# $\sin(2\beta + \gamma)$ from $B_d^0(t) \to D^{*\pm}\rho^{\mp}$: Theoretical Issues

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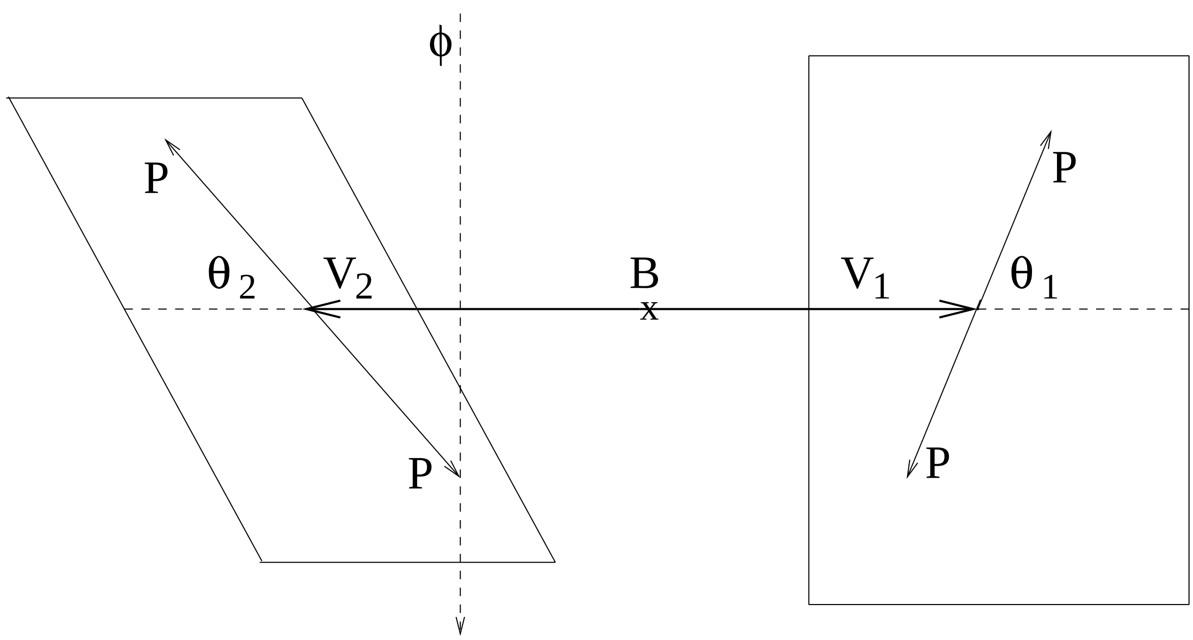
# Angular Analysis

Problem with indirect CP violation in $B \to V_1 V_2$ decays: $V_1 V_2$ not a CP eigenstate.

$V_1 V_2$ has 3 helicity amplitudes: $A_0$, $A_\parallel$ (CP-even), $A_\perp$ (CP-odd):

Well known: can separate helicity amplitudes using a (time-dependent) angular analysis. Can then measure indirect CP asymmetries in each helicity state.




Note: $\phi$ is the angle between the two decay planes.

The time-integrated differential decay rate contains 6 angular terms:

$$
\begin{aligned}
\frac{d\Gamma}{d\cos\theta_1 d\cos\theta_2 d\phi} \quad \sim \quad & |A_0|^2 \cos^2\theta_1 \cos^2\theta_2 \\
& + \frac{|A_\perp|^2}{2} \sin^2\theta_1 \sin^2\theta_2 \sin^2\phi \\
& + \frac{|A_\parallel|^2}{2} \sin^2\theta_1 \sin^2\theta_2 \cos^2\phi \\
& + \frac{\mathrm{Re}(A_0 A_\parallel^*)}{2\sqrt{2}} \sin 2\theta_1 \sin 2\theta_2 \cos\phi \\
& - \frac{\mathrm{Im}(A_\perp A_0^*)}{2\sqrt{2}} \sin 2\theta_1 \sin 2\theta_2 \sin\phi \\
& - \frac{\mathrm{Im}(A_\perp A_\parallel^*)}{2} \sin^2\theta_1 \sin^2\theta_2 \sin 2\phi \ .
\end{aligned}
$$

Note: angular analysis does more than simply separate helicities. One can also measure interference of different helicities. Gives much more information.

Also: don't have to fit to all 6 terms. By judicious integrations over angles, can eliminate certain terms. E.g. integrate over $\theta_1$ and $\theta_2 \Longrightarrow$ only $\mathrm{Im}(A_\perp A_\parallel^*)$ remains among the cross terms.

Now consider a final state $f = V_1 V_2$ to which both $B_d^0$ and $\bar{B}_d^0$ can decay:

$$
\begin{aligned}
A_\lambda &\equiv Amp(B_d^0 \to f)_\lambda \ , \\
A'_\lambda &\equiv Amp(\bar{B}_d^0 \to f)_\lambda \ ,
\end{aligned}
$$

$\lambda$ takes the values $\{0, \|, \perp\}$.

The time-dependent decay amplitude takes the form

$$
\begin{aligned}
\Gamma(B^0(t) \to f) \ \sim \ & \sum_{\lambda \le \sigma} \Big( \Lambda_{\lambda\sigma} + \Sigma_{\lambda\sigma} \cos(\Delta M t) \\
& \qquad - \rho_{\lambda\sigma} \sin(\Delta M t) \Big) g_\lambda g_\sigma \ ,
\end{aligned}
$$

where the $g_\lambda, \ g_\sigma$ are angular functions.
$\Lambda_{\lambda\sigma}$, $\Sigma_{\lambda\sigma}$, $\rho_{\lambda\sigma}$ each depend on the $A_\lambda$ and $A'_\lambda$.

By performing a time-dependent angular analysis (similar to that on the previous slide), it is possible to obtain each of $\Lambda_{\lambda\sigma}$, $\Sigma_{\lambda\sigma}$, $\rho_{\lambda\sigma}$.

Note: a full description of the time-dependent decay rate requires the measurement of 18 parameters. However, if a full description is not necessary, can eliminate some parameters by integrating over some angles.

## $B_d^0(t) \to D^{*\pm}\rho^{\mp}$

Now consider the decays $B_d^0(t) \to D^{*\pm}\rho^{\mp}$: pure tree, no penguin contributions.

$B_d^0 \to D^{*-}\rho^{+}$: $V_{cb}^* V_{ud}$, $B_d^0 \to D^{*+}\rho^{-}$: $V_{ub}^* V_{cd}$:

$$
\begin{aligned}
A_\lambda &\equiv Amp(B_d^0 \to D^{*-}\rho^{+})_\lambda = a_\lambda \, e^{i\delta_\lambda^a} \;, \\
A'_\lambda &\equiv Amp(\bar{B}_d^0 \to D^{*-}\rho^{+})_\lambda = b_\lambda \, e^{i\delta_\lambda^b} \, e^{-i\gamma} \;, \\
\bar{A}'_\lambda &\equiv Amp(B_d^0 \to D^{*+}\rho^{-})_\lambda = b_\lambda \, e^{i\delta_\lambda^b} \, e^{i\gamma} \;, \\
\bar{A}_\lambda &\equiv Amp(\bar{B}_d^0 \to D^{*+}\rho^{-})_\lambda = a_\lambda \, e^{i\delta_\lambda^a} \;.
\end{aligned}
$$

Note: these decays are described by 12 theoretical parameters: 3 $a_\lambda$'s, 3 $b_\lambda$'s, 5 relative strong phases, 1 weak phase. (As we will see, the weak phase $2\beta + \gamma$ is probed here.)

Need to measure 2 angular time-dependent decay rates:

$$\Gamma(B_d^0(t) \to D^{*-}\rho^+) \sim \sum_{\lambda \le \sigma} \Big( \Lambda_{\lambda\sigma} + \Sigma_{\lambda\sigma} \cos(\Delta M t) - \rho_{\lambda\sigma} \sin(\Delta M t) \Big) g_\lambda g_\sigma \ ,$$

$$\Gamma(B_d^0(t) \to D^{*+}\rho^-) \sim \sum_{\lambda \le \sigma} \Big( \bar{\Lambda}_{\lambda\sigma} + \bar{\Sigma}_{\lambda\sigma} \cos(\Delta M t) - \bar{\rho}_{\lambda\sigma} \sin(\Delta M t) \Big) g_\lambda g_\sigma \ .$$

$\Lambda_{\lambda\sigma}$, $\Sigma_{\lambda\sigma}$, $\rho_{\lambda\sigma}$ each depend on the $A_\lambda$ and $A'_\lambda$,

$\bar{\Lambda}_{\lambda\sigma}$, $\bar{\Sigma}_{\lambda\sigma}$, $\bar{\rho}_{\lambda\sigma}$ each depend on the $\bar{A}_\lambda$ and $\bar{A}'_\lambda$.

Note: these two decays appear to be described by 36 observables. However, only 12 are independent (because there are only 12 theoretical parameters).

$\exists$ enough information to solve for all theoretical parameters. The weak phase appears only in $\rho_{\lambda\sigma}$ and $\bar{\rho}_{\lambda\sigma}$. Because of the mixing phase $\exp(-2i\beta)$, it is the weak phase $2\beta + \gamma$ which is probed. Explicit solutions are possible (though expressions are complicated).

Method relies on the measurement of the interference terms between different helicities. However, only two helicities are required. Thus, use observables involving the two largest helicity amplitudes. E.g. if $\lambda = 0, \perp$ are largest, find

$$\sin^2(2\beta+\gamma) = f(\Lambda_{\lambda\lambda}, \Sigma_{\lambda\lambda}, \Lambda_{\perp 0}, \Sigma_{\perp 0}, \rho_{\lambda\lambda}, \bar{\rho}_{\lambda\lambda}, \rho_{\perp 0}, \bar{\rho}_{\perp 0}) \ .$$

Note: only 12 observables involved (the $\bar{\Lambda}$'s are proportional to the $\Lambda$'s, and similarly for the $\bar{\Sigma}$'s and $\Sigma$'s).

Are there ways to simplify the experimental analysis?

As noted earlier, a full angular analysis is not necessary. The method requires measurements of only two helicities.

Also, this method requires a fit to 12 observables. One can get partial information about these observables elsewhere:

- The (easier) time-integrated angular analysis measures the $\Lambda_{\lambda\sigma} + \Sigma_{\lambda\sigma}$. This might help constrain the time-dependent fit.
- $B^+ \to D^{*+}\rho^0$ is related to $B_d^0 \to D^{*+}\rho^-$. Angular analysis of $B^\pm \to D^{*\pm}\rho^0$ gives info about $A'_\lambda$ and $\bar{A}'_\lambda$, which are related to $\Lambda$'s and $\Sigma$'s. Again, this knowledge might help constrain the time-dependent fit.
  (Not true for $B^+ \to \bar{D}^{*0}\rho^+$ and $B_d^0 \to D^{*-}\rho^+$ since former has both $T$ and $C$ contributions, while latter is purely $T$.)

Note: two interfering amplitudes $B_d^0 \to D^{*-}\rho^+$ and $\bar{B}_d^0 \to D^{*-}\rho^+$ are of very different size: ratio is

$$r \equiv \left| \frac{A(B_d^0 \to D^{*-}\rho^+)}{A(\bar{B}_d^0 \to D^{*-}\rho^+)} \right| \sim \left| \frac{V_{ub}^* V_{us}}{V_{cb}^* V_{cs}} \right| \simeq 2\% \ .$$

Thus, need to measure things to better than 2%. This may not be easy.

It may therefore be better to consider decays in which two interfering amplitudes are more similar in size. These are: $B_d^0 \to \bar{D}^{*0} K^{*0}, D^{*0} K^{*0}$ and $\bar{B}_d^0 \to D^{*0} \bar{K}^{*0}, \bar{D}^{*0} \bar{K}^{*0}$, in which both $K^{*0}$ and $\bar{K}^{*0}$ decay to the same state $K_S \pi^0$. In this case the two amplitudes are much more equal in size, leading to a large asymmetry of $\sim 40\%$. The disadvantage is that the BR's for such Cabibbo-suppressed decays are much smaller than those for $B_d^0/\bar{B}_d^0 \to D^{*\pm}\rho^\mp$.

# Other Methods

1. $B_d^0(t) \to D^{(*)\pm}\pi^\mp$ [I. Dunietz, PLB 427, 179, 1998]:

The time-dependent rates for $B_d^0(t) \to D^{(*)\pm}\pi^\mp$ allow one to extract the following quantities:

$$C \equiv \frac{1-r^2}{1+r^2} \quad , \qquad S^\pm \equiv \frac{2r}{1+r^2}\sin(2\beta+\gamma\pm\delta) \ ,$$

where $r$ is the ratio of $b \to u$ and $b \to c$ amplitudes, and $\delta$ is a strong phase. $r$ can be extracted from the measurement of $C$, which allows $\sin(2\beta+\gamma)$ to be obtained from $S^\pm$.

Method is clean. Problem: in order to extract $r$, $C$ has to be measured to a precision of $r^2 \sim 10^{-4}$. This is virtually impossible.

BaBar: assumes $r$ can be estimated from

$$r = \tan\theta_C \sqrt{\frac{\mathcal{B}(B_d^0 \to D_s^{*+}\pi^-)}{\mathcal{B}(B_d^0 \to D^{*-}\pi^+)}} \frac{f_{D^*}}{f_{D_s^*}} \ .$$

Taking the ratio of decay constants from lattice, they find

$$r = 0.017^{+0.005}_{-0.007} \ .$$

The above error includes an assigned 30% theoretical error. Procedure reasonable. But cannot be used to give a precise measurement of $\sin(2\beta+\gamma)$.

2. $B_d^0(t) \to D^{*\pm} a_1^{\mp}$ [M. Gronau, D. Pirjol and D. Wyler, PRL 90, 051801, 2003]:

$a_1$ detected via $a_1^+ \to \rho^0 \pi^+ \to \pi^+ \pi^- \pi^+$. Analysis similar to $B_d^0(t) \to D^{*\pm} \rho^{\mp}$ except: because one has a 3-body decay, there are more angular variables. Allows more information to be extracted — get $\sin(2\beta + \gamma)$ **and** $\cos(2\beta + \gamma)$. Resolves twofold ambiguity in extraction of CP angles.

Analysis more complicated than $B_d^0(t) \to D^{*\pm} \rho^{\mp}$: Dalitz plot, more parameters to fit. However, payoff (resolution of discrete ambiguity) is significant.

3. $B_d^0(t) \to D^{\pm} K_S \pi^{\mp}$ [R. Aleksan and T.C. Petersen, hep-ph/0307371]:

Idea: decays $B_d^0 \to D^{\pm} K_S \pi^{\mp}$ are both governed by color-allowed tree amplitudes. Both diagrams are of similar size $\Longrightarrow$ interference is sizeable. Analyze 3-body decays using Dalitz plot. Measure $C$ and $S^{\pm}$ from time-dependent decay rates for particular point in Dalitz plot. Measure $r$ (ratio of $b \to u$ and $b \to c$ amplitudes) and extract $\sin(2\beta + \gamma)$.

Downside: assumes significant nonresonant 3-body decays. If this is small, method does not work. Also, requires a fairly complicated analysis (Dalitz plots) $\Longrightarrow$ not clear that one can obtain a good precision on $\sin(2\beta + \gamma)$, even with $500 \text{ fb}^{-1}$.

Reminder: if one wants two interfering amplitudes of similar size, it might be useful to consider the $B_d^0(t) \to D^{*\pm} \rho^{\mp}$ analysis applied to the Cabibbo-suppressed 2-body decays $B_d^0 \to \bar{D}^{*0} K^{*0}, D^{*0} K^{*0}$; $\bar{B}_d^0 \to D^{*0} \bar{K}^{*0}, \bar{D}^{*0} \bar{K}^{*0}$.

# Questions

1. Will the need for theory assumptions go away completely if we try the LSS idea for angular analysis in $B_d^0(t) \to D^* \rho$?

- The method of measuring $B_d^0(t) \to D^{*\pm} \rho^{\mp}$ to obtain $\sin^2(2\beta + \gamma)$ is theoretically robust. No theoretical assumptions (e.g. SU(3), etc.) needed. Only assumption is that a single weak amplitude contributes to each decay.

2. The LSS method assumes that all $B_d^0 \to D^* \pi^- \pi^0$ is $B_d^0 \to D^* \rho^-$, but we have reasons to believe that there is some non-resonant component in $B_d^0 \to D^* \pi^- \pi^0$. What guidance do you have on dealing with non-resonant contribution in the angular and time-dependent analysis?

- The method assumes that the $\pi\pi$ pair comes from a vector $\rho$-meson. This is reflected in the specific form of the angular functions $g_\lambda$. Nonresonant contributions will not have the same angular form. It will therefore be necessary to perform a fit and keep only the data which characterize a vector decay. I believe BaBar has done this in determining the $B \to D^* \rho$ branching ratios.

3. What advances are needed for theory to reliably provide $r$ (the ratio of $b \to u / b \to c$ amplitudes)?

- Difficult to say. We need better ways to extract $|V_{ub}|$, I guess. However, knowledge of $r$ is not needed in the $B \to D^* \rho$ method. In fact, $r$ can be *derived* from the data, along with the other theoretical quantities. Note also: the necessary precision here is $O(r)$, and not $O(r^2)$, as in $B_d^0(t) \to D^{(*)^\pm} \pi^\mp$.

4. What other associated decay modes, with a sample of 500 fb$^{-1}$, should we look for which can aid in constraining the theoretical ambiguities and assumptions (e.g. modes that give a sense of the strength of $W$-exchange and final-state rescattering)?

- As stated before, in the $B \to D^* \rho$ method, the theory input is minimal. There are no $W$-exchange contributions to these decays, and any rescattering is expected to be negligible. The solution does have a discrete ambiguity — the sign of $\sin(2\beta + \gamma)$ is not determined — but this can in principle be removed by looking at $B \to D^* a_1$.

5. Can one "recycle" information gained from an angular analysis of $B \to D^* \rho$ into $B \to D^{(*)} \pi$ time-dependent analyses? Are the $r$'s likely to be the same?

- $B \to D^* \rho$ is not related in any simple way to $B \to D\pi$. Comparing $B \to D^* \rho$ and $B \to D^* \pi$, I would expect the $r$'s to be comparable (they're equal in naive factorization). Given the value of $r$ from $B \to D^* \rho$, it would be interesting to compare the values of $\sin(2\beta + \gamma)$ obtained from $B_d^0(t) \to D^{*\pm} \rho^{\mp}$ and $B_d^0(t) \to D^{(*)^{\pm}} \pi^{\mp}$.