$\sin 2\beta$

Yuval Grossman

My task

- Overview the SM calculations of $S(f) - S(\psi K_S)$.
  - Explain the "model-independent" SU(3) approach.
  - What is the estimate for $\pi^0 K_S$?
- What is the set of new physics scenarios that can explain a large value of $S(\phi K_S) - S(\psi K_S)$?
- Give an example of a theory that could explain a large deviation in ϕK_S, but not in $\eta' K_S$, $K^+K^-K_S$, or $\pi^0 K_S$.
  - In this scenario, what other decay modes would show a corresponding deviation from $\sin 2\beta$?
  - Are there effects expected in the rates, asymmetries or polarizations of other charmless decays?
- Which decay modes would you like experimenters to look at that we are not currently exploring?

Estimating $S(f) - S(\psi K_S)$

The problem with $b \to sq\bar{q}$ decays

$$A = \underbrace{\overset{[\lambda^2]}{V_{cb}V_{cs}^*}}_{\text{dominant contribution}} [P_c - P_t + T_{c\bar{c}s}] + \underbrace{\overset{[\lambda^4]}{V_{ub}V_{us}^*}}_{\text{suppressed by } \lambda^2} [P_u - P_t + T_{u\bar{u}s}]$$

- We often use $A = P + T \qquad T = |T|e^{i\gamma}$
- When $T \ll P$ then A is dominated by a single phase
  - $C_{s\bar{q}q} \approx 0$
  - $S_{s\bar{q}q} \approx S_{\psi K}$
- With new physics: $S_{s\bar{q}q} \neq S_{\psi K}$ and $C_{s\bar{q}q} \neq 0$ possible

How large are the subleading effects in the SM?

Definitions

$$A_f \equiv A(B^0 \to f) = V_{cb}^* V_{cs}\, a_f^c + V_{ub}^* V_{us}\, a_f^u = V_{cb}^* V_{cs}\, a_f^c\,(1 + \xi_f)$$

$$\xi_f \equiv \frac{V_{ub}^* V_{us}}{V_{cb}^* V_{cs}}\,\frac{a_f^u}{a_f^c}\,, \qquad \left|\frac{V_{ub}^* V_{us}}{V_{cb}^* V_{cs}}\right| = \mathcal{O}(\lambda^2)\,, \qquad \delta_f = \arg\frac{a_f^u}{a_f^c}$$

- $S_f - \sin 2\beta \approx 2\cos 2\beta\, \sin\gamma\, \cos\delta_f\, |\xi_f|$
- $C_f \approx -2\sin\gamma\, \sin\delta_f\, |\xi_f|$

We like to get the value of $|\xi_f|$

Getting ξ_f

We concentrate on $b \to s$ modes where we expect $\xi_f \ll 1$

$$B \to \phi K_S \quad B \to \eta' K_S \quad B \to \pi^0 K_S \;\; \ldots$$

Two ways

- Calculate (with experimental input)
- Using flavor symmetries

Trying to get as much information as possible. The two ways are both important

An example: $B \to \eta' K_S$

Beneke and Neubert

$|\xi_{\eta' K_S}|$ was calculated using QCD factorization (BBNS)

$$|\xi_{\eta' K_S}| \approx 0.06 - 0.09$$

- The spread is due to model dependence
- The strong phase was also calculated and found to be small

SU(3) relations: example

$$B \to \pi^0 K \ (b \to sq\bar{q}) \qquad B_s \to \pi^0 \bar{K} \ (b \to dq\bar{q})$$

Using U-spin ($\lambda \sim 0.22$)

$$A(B \to \pi^0 K) \equiv T + P \qquad A(B_s \to \pi^0 \bar{K}) = \lambda^{-1} T + \lambda P$$

For simplicity we assume

$$A(B \to \pi^0 K) \approx P \qquad A(B_s \to \pi^0 \bar{K}) \approx \lambda^{-1} T$$

We get

$$\xi = \frac{T}{P} \approx \lambda \frac{A(B_s \to \pi^0 \bar{K})}{A(B \to \pi^0 K)} \approx \lambda \sqrt{\frac{\Gamma(B_s \to \pi^0 \bar{K})}{\Gamma(B \to \pi^0 K)}}$$

SU(3) relations

- For $b \to q\bar{q}s$ transitions

$$A_f = V_{cb}^* V_{cs}\, a_f^c + V_{ub}^* V_{us}\, a_f^u = V_{cb}^* V_{cs}\, a_f^c\,(1 + \xi_f)$$

- For $b \to q\bar{q}d$ transitions

$$A_{f'} = V_{cb}^* V_{cd}\, b_{f'}^c + V_{ub}^* V_{ud}\, b_{f'}^u = V_{ub}^* V_{ud}\, b_{f'}^u\,(1 + \lambda^2 \xi_{f'}^{-1})$$

SU(3) gives relations among a_f^q and $b_{f'}^q$

$$a_f^u = \sum_{f'} x_{f'}\, b_{f'}^u$$

$x_{f'}$ are CG coefficients.

SU(3) relations

- The branching ratios $\mathcal{B}(f)$ constrain a_f^c and $b_{f'}^u$

$$|\xi_f| \equiv \left| \frac{V_{ub}^* V_{ud}}{V_{cb}^* V_{cs}} \frac{b_{f'}^u}{a_f^c} \right| \sim \lambda \sqrt{\frac{\mathcal{B}(f')}{\mathcal{B}(f)}}$$

  This can be use to get our "best estimate" for $|\xi_f|$

- Combining $SU(3)$ and experimental data gives

$$\hat{\xi}_f \equiv \left| \frac{\xi_f + \lambda^2}{1 - \xi_f} \right| \leq \lambda \sum_{f'} |x_{f'}| \sqrt{\frac{\mathcal{B}(f')}{\mathcal{B}(f)}}$$

  This is used to get exact bounds (in the SU(3) limit)

Results: $B \to \pi^0 K_S$

- SU(3) relation

$$a(\pi^0 K^0) = b(\pi^0\pi^0) + b(K^+K^-)/\sqrt{2}$$

- Data: $\mathcal{B}(B^0 \to \pi^0 K^0) = (11.92 \pm 1.44) \times 10^{-6}$

$$\mathcal{B}(B^0 \to \pi^0 \pi^0) = (1.89 \pm 0.46) \times 10^{-6}$$

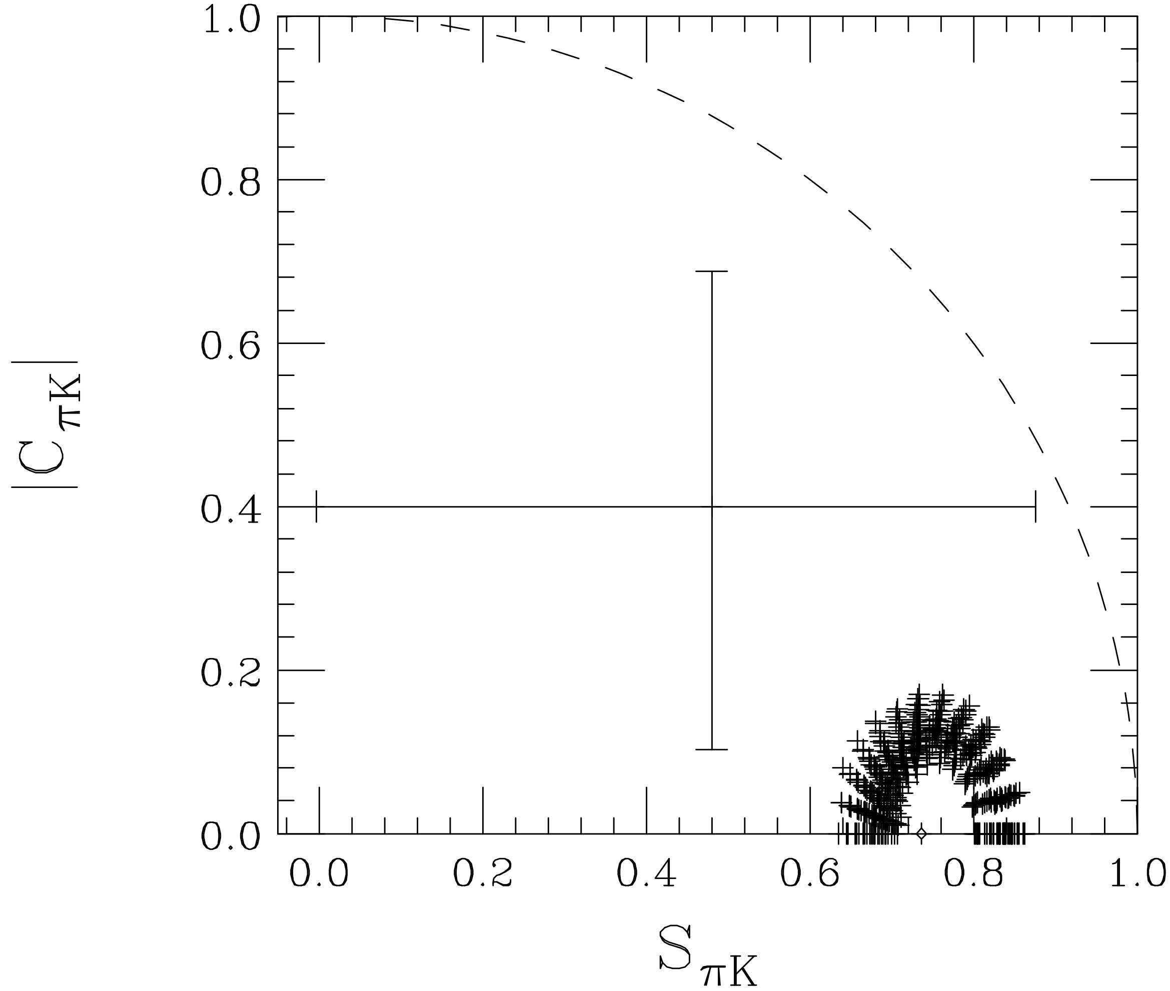
$$\mathcal{B}(B^0 \to K^+ K^-) < 0.6 \times 10^{-6}$$

- We get

$$\xi \sim 0.08, \quad \hat{\xi} < 0.13, \quad |S_{\pi K} - \sin 2\beta| < 0.19, \quad |C_{\pi K}| < 0.26$$

- We expect $\mathcal{B}(B^0 \to K^+K^-)$ to be very small. Neglecting it we get stronger bounds

$B \to \pi^0 K_S$



- Neglecting $B^0 \to K^+ K^-$

Results: $B \to \eta' K_S$

More complicated SU(3) relation

$$a(\eta' K^0) = \frac{s^2 - 2c^2}{2\sqrt{2}} b(\eta'\pi^0) - \frac{3sc}{2\sqrt{2}} b(\eta\pi^0) + \frac{\sqrt{3}s}{4} b(\pi^0\pi^0) - \frac{\sqrt{3}s(s^2+4c^2)}{4} b(\eta'\eta') + \frac{3\sqrt{3}sc^2}{4} b(\eta\eta) + \frac{\sqrt{3}c(2c^2-s^2)}{2\sqrt{2}} b(\eta\eta')$$

$s \equiv \sin\theta_{\eta\eta'}$, $c \equiv \cos\theta_{\eta\eta'}$

Results: $B \to \eta' K_S$

Which gives

$$\hat{\xi}_{\eta' K_S} < \lambda \Bigg[0.59\sqrt{\frac{\mathcal{B}(\eta'\pi^0)}{\mathcal{B}(\eta' K^0)}} + 0.33\sqrt{\frac{\mathcal{B}(\eta\pi^0)}{\mathcal{B}(\eta' K^0)}} + 0.14\sqrt{\frac{\mathcal{B}(\pi^0\pi^0)}{\mathcal{B}(\eta' K^0)}} + 0.53\sqrt{\frac{\mathcal{B}(\eta'\eta')}{\mathcal{B}(\eta' K^0)}} + 0.38\sqrt{\frac{\mathcal{B}(\eta\eta)}{\mathcal{B}(\eta' K^0)}} + 0.96\sqrt{\frac{\mathcal{B}(\eta\eta')}{\mathcal{B}(\eta' K^0)}} \Bigg]$$

- No best estimate for $\xi_{\eta' K_S}$ yet
- Numerically $\hat{\xi}_{\eta' K_S} < 0.36$

Bound from charged mode

Similar relations hold for the charged modes

$$a(\eta' K^+) = b(\eta' \pi^+) + s\sqrt{\frac{3}{2}}b(\overline{K^0}K^+)$$

Using experimental data

$$\hat{\xi}_{\eta' K^+} < 0.09$$

- We have $a^c_{\eta' K^0} = a^c_{\eta' K^+}$, but $a^u_{\eta' K^0} \neq a^u_{\eta' K^+}$
- $a^u_{\eta' K^+}$ has a color-allowed tree diagram contribution
- $a^u_{\eta' K^0}$ is color-suppressed
- Dynamical assumption: $|a^u_{\eta' K^0}| \not> |a^u_{\eta' K^+}| \Rightarrow \hat{\xi}_{\eta' K_S} < 0.09$

$B \to \phi K_S$

For PV final state, even more complicated relations

$$a(\phi K^0) = \frac{1}{2}\left[b(\overline{K^{*0}}K^0) - b(K^{*0}\overline{K^0})\right] + \frac{1}{2}\sqrt{\frac{3}{2}}\left[cb(\phi\eta) - sb(\phi\eta')\right]$$
$$+ \frac{\sqrt{3}}{4}\left[cb(\omega\eta) - sb(\omega\eta')\right] - \frac{\sqrt{3}}{4}\left[cb(\rho^0\eta) - sb(\rho^0\eta')\right]$$
$$+ \frac{1}{4}b(\rho^0\pi^0) - \frac{1}{4}b(\omega\pi^0) - \frac{1}{2\sqrt{2}}b(\phi\pi^0)$$

- No bound on $\hat{\xi}_{\phi K_S}$ with present data
- No best estimate for $\xi_{\phi K_S}$ with present data
- Charged modes: $a(\phi K^+) = b(\phi\pi^+) + b(\overline{K^{*0}}K^+)$
- Dynamical assumption $|a^u_{\phi K^0}| \not> |a^u_{\phi K^+}| \quad \Rightarrow \hat{\xi}_{\phi K_S} < 0.25$

Comments on SU(3)

- Similar analysis for $B \to K^+K^-K_S$; more complicated and not covered here
- SU(3) relations are most useful for simple relations
- SU(3) and U spin are the same
- Since we use SU(3) there are large, $O(30\%)$, corrections. They can be larger or smaller in specific cases
- Bottom line: Large deviations from the SU(3) bounds are signals for new physics

New physics

New physics in loop

- Since $B \to \eta' K_S$, $B \to \phi K_S$ and $B \to \pi K_S$ are dominate by the one loop diagram in the SM we expect large new physics effects
- Due to different hadronic matrix elements we expect the shift from $\sin 2\beta$ to be different in these modes
- $B \to \psi K_S$ is a CKM favored tree level decay in the SM $\Rightarrow$ we expect small new physics effects

$$\Downarrow$$

NP in $b \to s\bar{q}q$ generally gives $S_{\psi K_S} \neq S_{\pi K_S} \neq S_{\phi K_S} \neq S_{\eta' K_S}$

Getting a shift only in $B \to \phi K_S$

Kagan

While no indication, still we ask: Can we get

$$S_{\phi K_S} \neq S_{\psi K_S} \text{ with } S_{\pi K_S} = S_{\eta' K_S} = S_{\psi K_S}$$

- $B \to \phi K_S$ is parity conserving while $B \to \eta' K_S$ is parity violating
- Parity conserving new physics in $b \to s$ penguins only affect $B \to \phi K_S$
- Generically, new physics models are not parity conserving
- Supersymmetric $SU(2)_L \times SU(2)_R \times \text{Parity}$ is an example of an approximate parity conserving new physics model

Opposite chirality

Extensions of SM often include opposite chirality operators related by parity ($V-A \leftrightarrow V+A$).

Example: QCD Penguin operators

$$
\begin{array}{ll}
\underline{\text{SM Chirality}} & \underline{\text{Opposite Chirality}} \\
Q_{3,5} = (\bar{s}b)_{V-A}\,(\bar{q}q)_{V\mp A} & \longrightarrow \tilde{Q}_{3,5} = (\bar{s}b)_{V+A}\,(\bar{q}q)_{V\pm A} \\
Q_{4,6} = (\bar{s}_i b_j)_{V-A}\,(\bar{q}_j q_i)_{V\mp A} & \longrightarrow \tilde{Q}_{4,6} = (\bar{s}_i b_j)_{V+A}\,(\bar{q}_j q_i)_{V\pm A}
\end{array}
$$

Opposite chiralities and decay amplitudes

- Effective Hamiltonian for $\Delta S = 1$ Decays:

$$\mathcal{H}_{\rm eff} \propto \sum_i C_i Q_i + \tilde{C}_i \tilde{Q}_i$$

- Under Parity, $Qi \leftrightarrow \tilde{Q}_i \;\Rightarrow$ final state, f, with parity P_f

$$\langle f|Q_i|B\rangle = (-1)^{P_B}(-1)^{P_f}\langle f|\tilde{Q}_i|B\rangle$$
$$\Rightarrow\; A_i(B \to f) \propto C_i - (-1)^{P_f}\tilde{C}_i$$

- In the SM $\tilde{C} = 0$, thus

$$A_i^{\rm NP}(B \to f) \propto C_i^{\rm NP} - (-1)^{P_f}\tilde{C}_i^{\rm NP}$$

- For P-invariant NP $A_i^{\rm NP} = 0$ for all P_f even states

Implication of opposite chiralities operators

- For decays with determined parity

$$A_i^{NP}(B \to PP) \propto C_i^{\rm NP}(\mu_b) - \tilde{C}_i^{\rm NP}(\mu_b)$$
$$A_i^{NP}(B \to VP) \propto C_i^{\rm NP}(\mu_b) + \tilde{C}_i^{\rm NP}(\mu_b)$$

- For $B \to VV$, $\perp$ transversity is P-odd; $0, \parallel$ are P-even:

$$A_i^{NP}(B \to VV)_{0,\parallel} \propto C_i^{\rm NP}(\mu_b) - \tilde{C}_i^{\rm NP}(\mu_b)$$
$$A_i^{NP}(B \to VV)_{\perp} \propto C_i^{\rm NP}(\mu_b) + \tilde{C}_i^{\rm NP}(\mu_b)$$

- For $B \to VA$, where $A \equiv$ Axial Vector:

$$A_i^{NP}(B \to VA)_{0,\parallel} \propto C_i^{\rm NP}(\mu_b) + \tilde{C}_i^{\rm NP}(\mu_b)$$
$$A_i^{NP}(B \to VA)_{\perp} \propto C_i^{\rm NP}(\mu_b) - \tilde{C}_i^{\rm NP}(\mu_b)$$

Examples

- P-even: $\eta' K$, $K\pi$, $K a_1$, $K_1\pi$, $(\phi K^*)_{0,\parallel}$, $(K^*\rho)_{0,\parallel}$,...
- P-odd: ϕK, $K^{*0}\pi$, $f_0 K$, $(\phi K^*)_{\perp}$, $(\phi K_1)_{0,\parallel}$,...

P-invariant new physics affects only the P-odd final states

- $S(f) - S(\psi K_S) \neq 0$
- $C(f) \neq 0$
- The effect is in general different in each of the P-odd modes
- Effects on polarization in the VV and VA modes
- Hard to see the effect on rates. Too large theoretical uncertainties

Left right symmetric new physics

It is not easy to naturally get $C_i = \tilde{C}_i$

- The SM is maximally parity violating
- Any model without a parity symmetry needs fine tuning
- Parity at the high scale must be broken
- Need to arrange that symmetry breaking effects are large for the SM sector and small for the NP sector
- Example: SUSY LRS model
  - SM: $m(W_L) \ll m(W_R)$
  - NP: $m(\tilde{q}_L) \approx m(\tilde{q}_R)$. Parity breaking via RGE only

Conclusions

Which decay modes are interesting

- Look for $b \to d$ decays to final states with η and η'
- Look for the CP asymmetries in the different transversity VV amplitudes
- Study P-odd modes, like $f_0 K$
- Measure the polarization of “all” VV and VA final states