Photon Spectrum in Inclusive Decays

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Postronium Decay

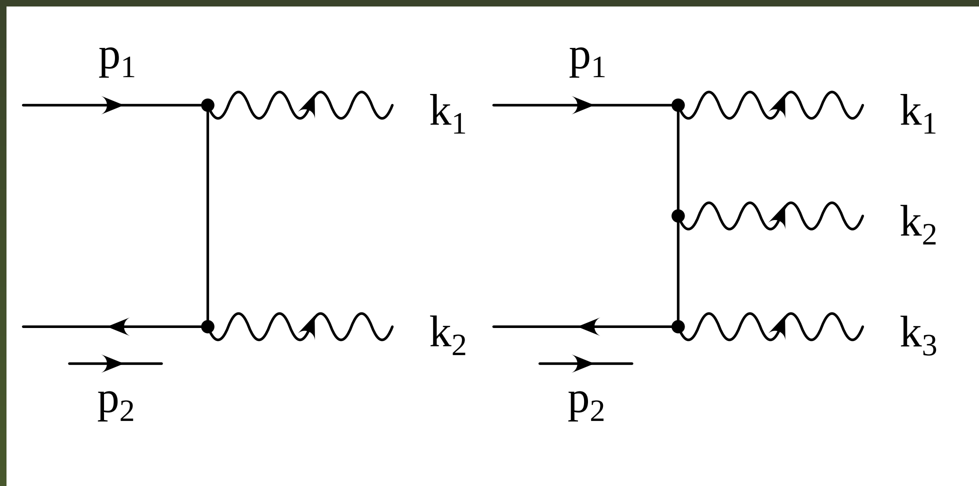
A.M. and P. Ruiz-Femenia, Phys Rev D69 (2004) 053003

Consider the decay of Ps as a radiative decay. In this case, the bound state is perturbative so everything is calculable.

See the transition from OPE to shape function to resonance spectrum explicitly.

Can also look at the photon spectrum in $\Upsilon \to \gamma + X$

Ps is an e^+e^- bound state (like Hydrogen). The $1S$ state can be spin zero (singlet, p-Ps) or spin one (triplet, o-Ps).



Look at $\frac{\mathrm{d}\Gamma}{\mathrm{d}E_\gamma}$ for o-Ps

Scales

Energy scales are

1. m: (OPE region)
2. $m\alpha^2$: comparable to the excitation energies of all states (shape function region)
3. $m\alpha^4$: comparable to the o-Ps–p-Ps mass difference (resonance region)

In B decays, the scales would be m, $\sqrt{m\Lambda_{\mathrm{QCD}}}$, and Λ_{QCD}.

Ore-Powell Spectrum

Compute the 3γ decay diagram ($x = 2E/(2m)$).

$$\frac{d\Gamma_{3\gamma}}{dx} = \frac{4m\alpha^6}{9\pi}\left[\frac{2-x}{x} + \frac{(1-x)\ x}{(2-x)^2} - \frac{2\,(1-x)^2\ \log(1-x)}{(2-x)^3} + \frac{2\ (1-x)\ \log(1-x)}{x^2}\right],$$

$$\Gamma_{3\gamma} = \frac{2\,(\pi^2-9)\,m\alpha^6}{9\pi}.$$

Perfectly fine series expansion in α.

Ore-Powell Spectrum

The low-energy photon spectrum results from the $x \to 0$ limit:

$$\frac{\mathrm{d}\Gamma_{3\gamma}}{\mathrm{d}x} = \frac{2m\alpha^6}{9\pi}\left[\frac{5x}{3} + \mathcal{O}(x^2)\right],$$

which vanishes linearly with the energy of the radiated photon.

Low's Theorem

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}E_\gamma} = \frac{A}{E_\gamma} + B + \mathcal{O}(E_\gamma),$$

A and B terms determined by pole graphs,

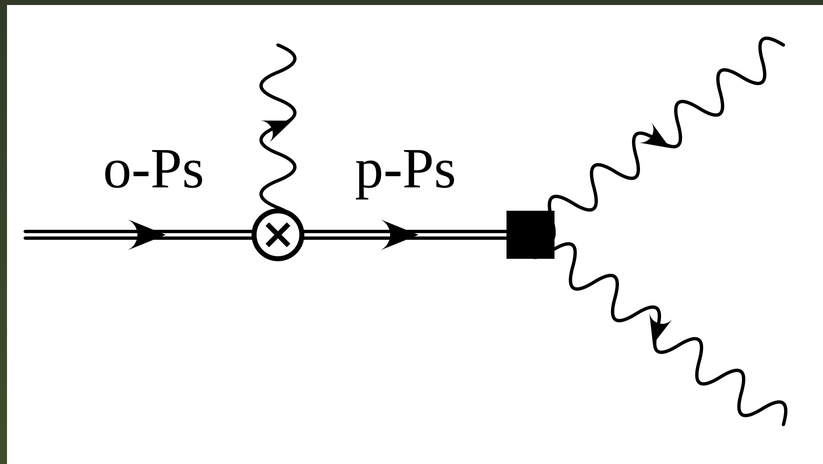
$$\text{o-Ps} \to \text{o-Ps} + \gamma, \qquad \text{o-Ps} \to 2\gamma$$

$$\frac{\mathrm{d}\Gamma_{\mathrm{oPs}\to 3\gamma}}{\mathrm{d}E_\gamma} \sim E_\gamma^3$$

as $E_\gamma \to 0$.

Resonance Region

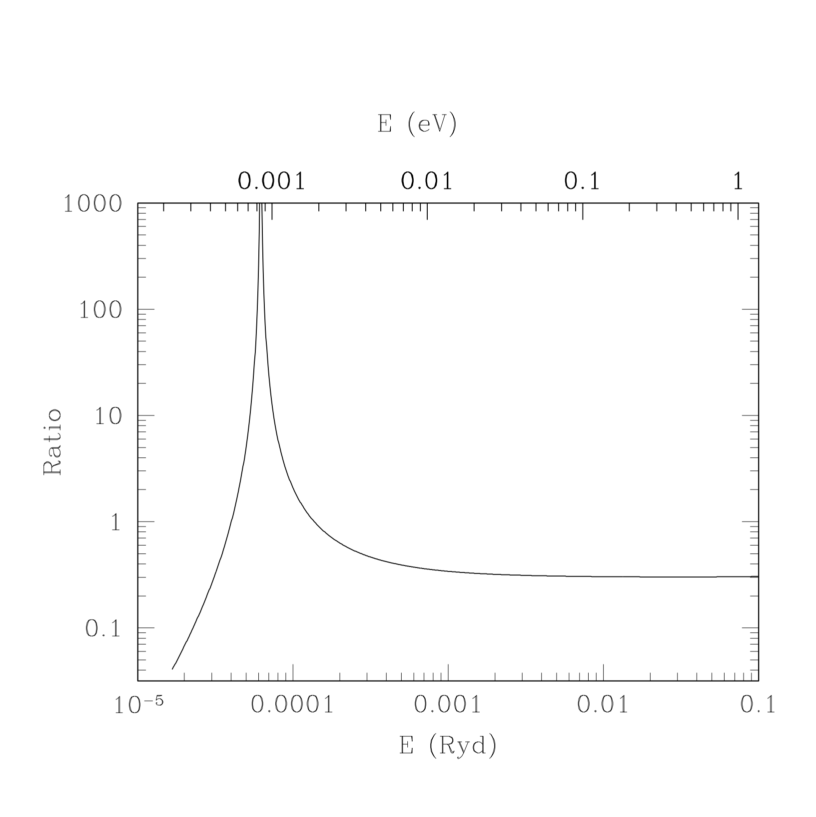
Energies of order $m\alpha^4$:



[The $1/E_\gamma$ term in Low's theorem would be from the o-Ps–o-Ps graph.]

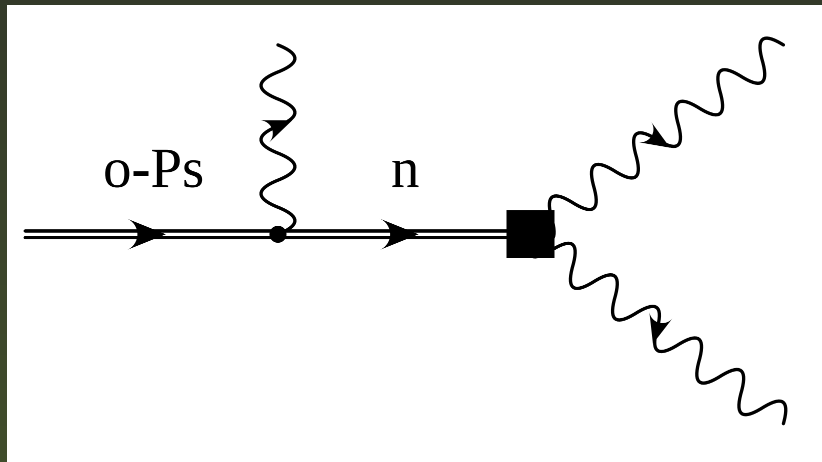
Graph depends on the magnetic dipole amplitude which $\propto E_\gamma$.
Need to know form factors.

Resonance Region



Shape Function Region

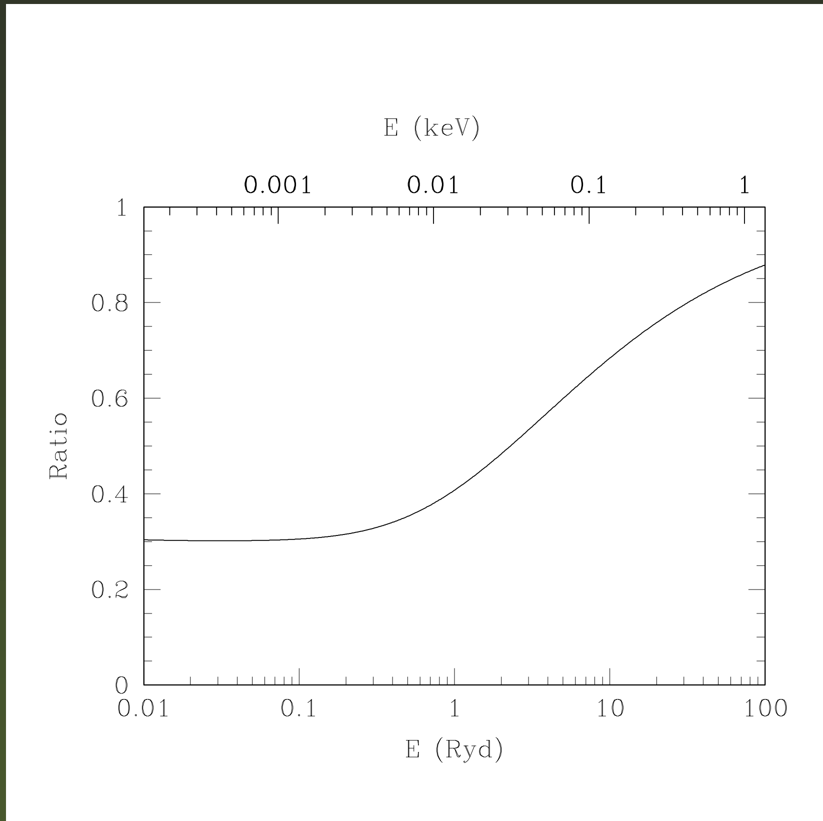
Energies of order $m\alpha^2$:



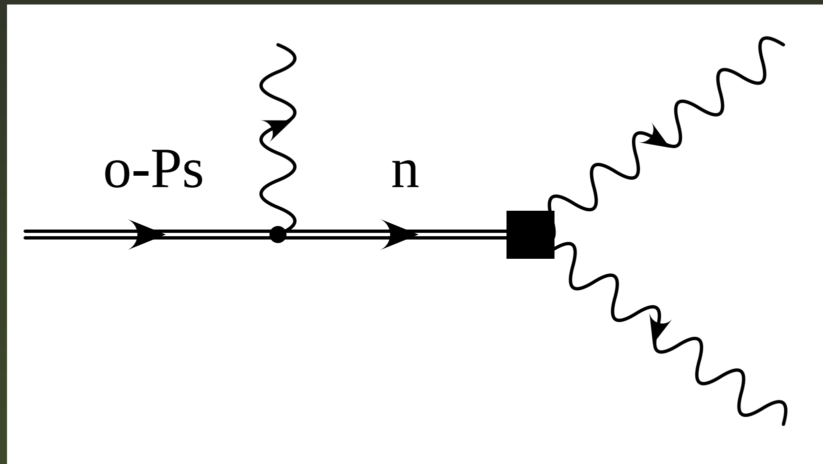
Sum over all the intermediate states n

Can do this explicitly.

Shape Function Region



The shape function is completely calculable.



Basically, one gets

$$\sum_n \frac{A_n}{E_\gamma + E_n - E_0}$$

OPE region: $E_\gamma \gg E_n - E_0$:

$$\sum_n \frac{A_n}{E_\gamma + E_n - E_0} \to \frac{1}{E_\gamma} \sum_n A_n \to \frac{A}{E_\gamma}$$

usually there are sum rules to give $\sum_n A_n = A$.

Shape function region: $E_\gamma \sim E_n - E_0$:

$$\sum_n \frac{A_n}{E_\gamma + E_n - E_0}$$

all states contribute $\to$ smooth function $S(E_\gamma)$.

Resonance region: only the nearest state is important:

$$\sum_n \frac{A_n}{E_\gamma + E_n - E_0} \to \frac{A_1}{E_\gamma + E_1 - E_0}$$

[e.g. $B^* - B$ mass difference is much smaller than that to excited B mesons.]