RadCorr Workshop [*March 14th 2005, San Diego*]

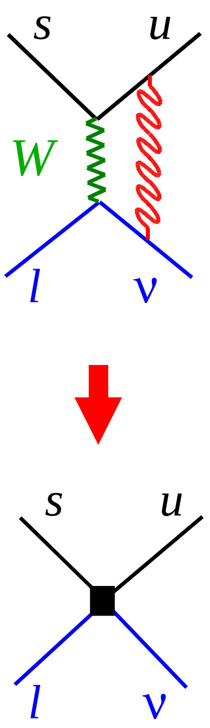
Radiative (e.m.) corrections in K (and B) decays

Gino Isidori [*INFN-Frascati*]

- Introduction
- Low's theorem & IR structure
- Soft-photon resummation
- UV structure of the effective theory
- The K_{l3} case

Introduction

In general, in weak *K* (& *B*) decays we can distinguish 3 basic types of e.m. corrections:



I. short-distance (UV) corrections to the 4-fermion eff. Hamiltonian

sizable [$\sim \alpha \log(\mu_{had}/M_W) \Rightarrow \delta\Gamma \sim$ few %]

well known

Marciano & Sirlin, '70- '80

$H_{eff}(\mu_{had})$

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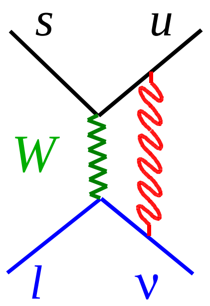

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II. matching between the 4-fermion H_{eff} and the meson theory (e.g. CHPT)
(+ hard-photon emission)

small [no large logs $\Rightarrow \delta\Gamma \sim$ few 10^{-3}]

model dependent [at least in the kaon case]

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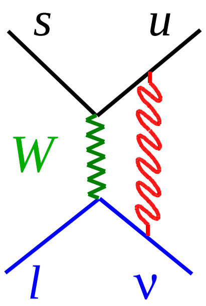

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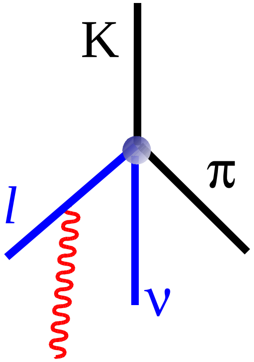

III. Bremsstrahlung & IR virtual corrections

sizable [~ $\alpha \log(M_K/m_e)$ ⇒ $\delta\Gamma$ ~ few %]

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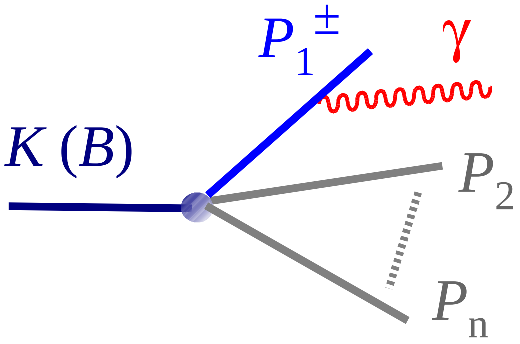
Cirigliano *et al.* '00-'04

➡ distortion of the decay distribution

Gatti '03 [KLOE]
Andre '04 [KTeV]

Low's theorem and IR singularities

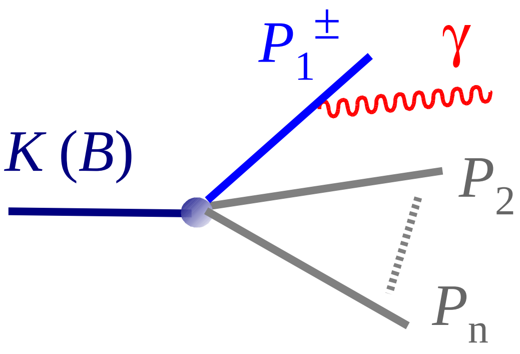


$$\frac{d\Gamma}{dE_\gamma} = \underbrace{b\,\frac{\Gamma_0}{E_\gamma} + c}_{\textit{bremsstrahlung}} + \underbrace{O(E_\gamma)}_{\textit{direct emission}}$$

The first two terms in the E_γ spectrum ($b\Gamma_0$ & c) are unambiguously determined in terms of the on-shell non-radiative amplitude

- b, c = $O(\alpha)$ coefficients
- Γ_0 = (unphysical) width of the non-radiative process ⇔ weak amplitude

Low's theorem and IR singularities



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E.g.: $\mathcal{A}(K \to 2\pi) = A_0 \quad \rightarrow \quad \mathcal{A}[K \to 2\pi\gamma(k)] = eA_0 \sum_i Q_i \frac{\varepsilon \cdot p_i}{k \cdot p_i}$

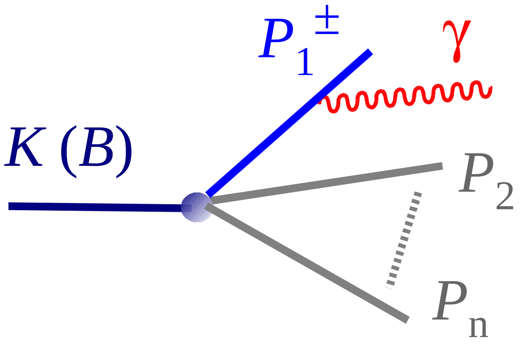
for $K^0 \to \pi^+\pi^-$:

$$\boxed{b = \frac{2\alpha}{\pi}\left[\frac{1+\beta^2}{2\beta}\ln\left(\frac{1+\beta}{1-\beta}\right) - 1\right] \approx 0.7\ \%}$$

$$\beta = \frac{p}{E} = \sqrt{1 - \frac{4m^2}{M^2}}$$

$c = 0$

Low's theorem and IR singularities



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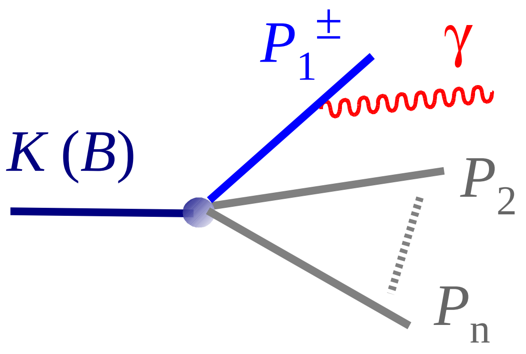
$$\boxed{b = \frac{2\alpha}{\pi}\left[\frac{1+\beta^2}{2\beta}\ln\left(\frac{1+\beta}{1-\beta}\right) - 1\right] \approx 3\,\%}$$

$$\beta = \frac{p}{E} = \sqrt{1 - \frac{4m^2}{M^2}}$$

$c = 0$

Low's theorem and IR singularities



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E.g.:

$$\mathcal{A}(K \to 3\pi) \ = \ A(s_{12}, s_{13}) \qquad\qquad s_{ij} = (p_i + p_j)^2$$

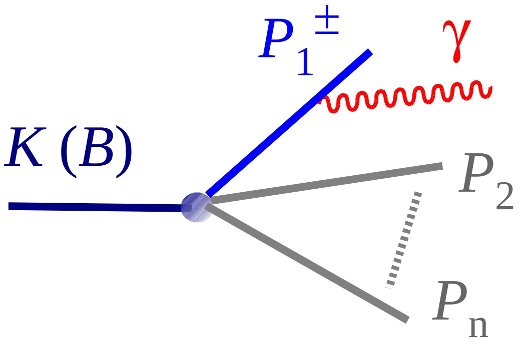
$$\mathcal{A}[K \to 3\pi\gamma(k)] \ = \ eA(s_{12}, s_{13})\Sigma + 2e\frac{\partial A(s_{12}, s_{13})}{\partial s_{12}}\Lambda_{12} + 2e\frac{\partial A(s_{12}, s_{13})}{\partial s_{13}}\Lambda_{13}$$

$$\Sigma \ = \ \sum_i Q_i \frac{\varepsilon \cdot p_i}{k \cdot p_i} \qquad \Lambda_{ij} \ = \ (Q_i\ k \cdot p_j - Q_j\ k \cdot p_i)\left(\frac{\varepsilon \cdot p_i}{k \cdot p_i} - \frac{\varepsilon \cdot p_j}{k \cdot p_j}\right)$$

In n > 2 body decays
both b & c ≠ 0 = *f* (kin. var.)

Convenient to express the decay ampl. in terms of neutral particle momenta (or the invariant mass of the sub-system which radiates)

Low's theorem and IR singularities



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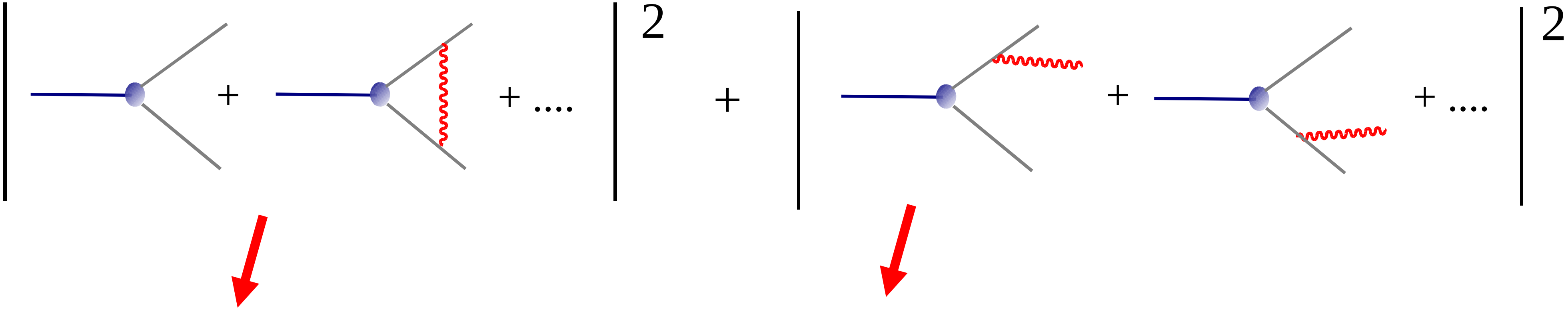
Second-dervatives lead to $O(E_\gamma)$ terms. Part of these terms can be conveniently encoded in the so-called *generalized bremsstrahlung* ampl. [D'Ambrosio et al. '96] ⇒ the corresponding direct emission does not depend from effective operaors contributing to $K \to 3\pi$.

At $O(\alpha)$ the $E_\gamma \to 0$ singularity can be integrated only introducing an IR cut off:

$$\int_{E_\gamma < E^{cut}} dE_\gamma \, \frac{d\Gamma}{dE_\gamma} = b\,\Gamma_0 \, \ln \frac{E^{cut}}{\mu_{IR}} \quad + \textit{regular terms}$$

whose dependence is canceled by the corresponding virtual corrections:



$$\Gamma_0 (1 + b \ln \frac{\mu_{IR}}{M/2}) \;+ \dots \qquad \Gamma_0 \, b \ln \frac{E^{cut}}{\mu_{IR}} \;+ \dots \quad = \quad \Gamma_0 (1 + b \ln \frac{2E^{cut}}{M}) \;+ \dots$$

Mass of the decaying particle

$$\Gamma[E^{cut}] = \Gamma[\, K \rightarrow X\,(\gamma)\,]\Big|_{E_\gamma < E^{cut}} = \Gamma_0 \,(1 + b \ln \frac{2E^{cut}}{M}) + \mathit{const} + O(E^{cut})$$

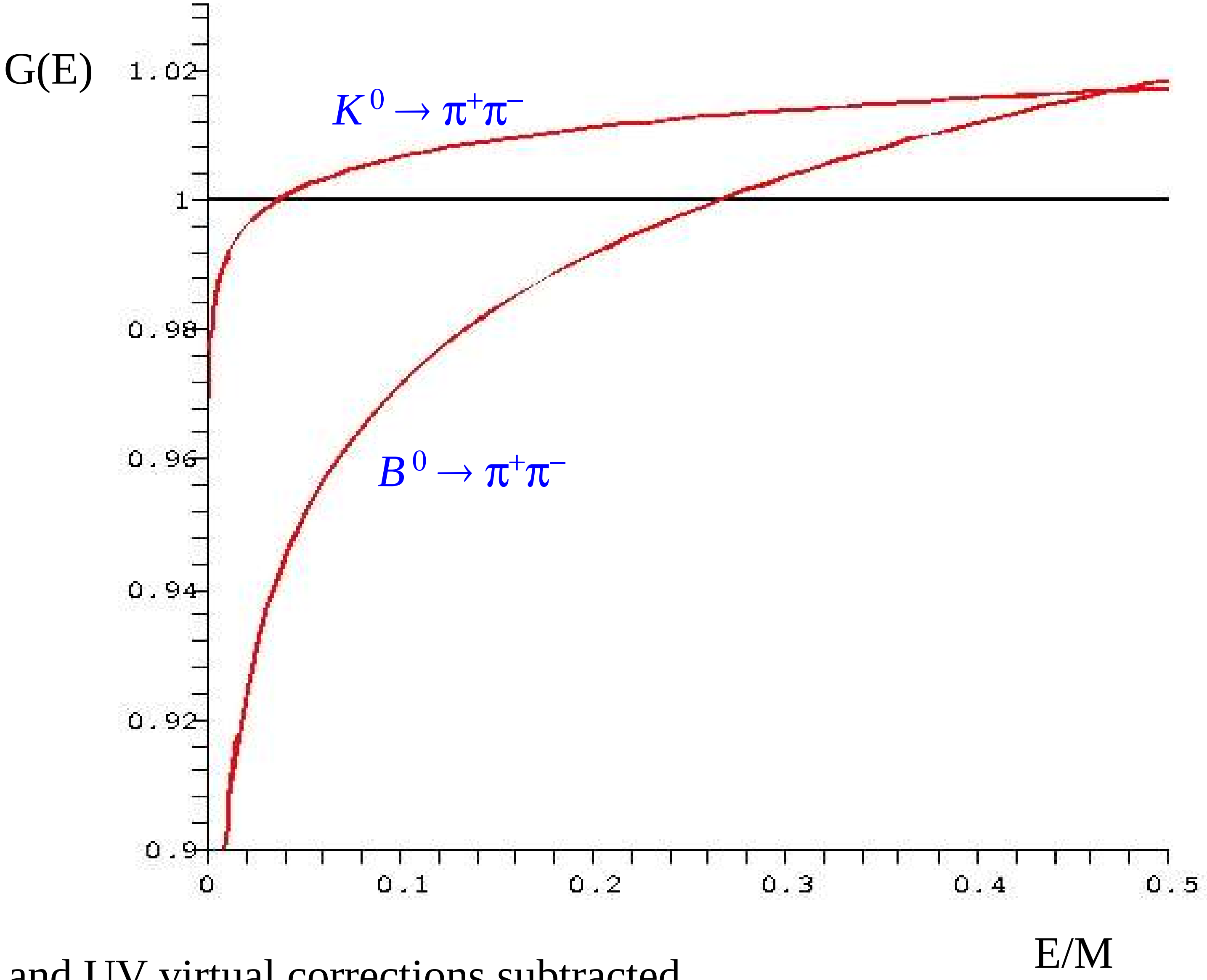
N.B.:

- Γ_0 is a theoretical quantity (not an observable)
- The overall size of the e.m. corrections in the relation between $\Gamma[E^{cut}]$ & Γ_0 is minimized in the *inclusive width* ($E^{cut} = E^{max} \approx M/2$):

$$\Gamma^{incl}[\, K \rightarrow X\,(\gamma)\,] = \Gamma_0\,[\, 1 + O(\alpha/\pi)\,]$$

- While b can be computed in a model-indep. way, the evaluation of the complete $O(\alpha/\pi)$ terms in Γ^{incl} requires a model-dependent calculation (weak dynamics)

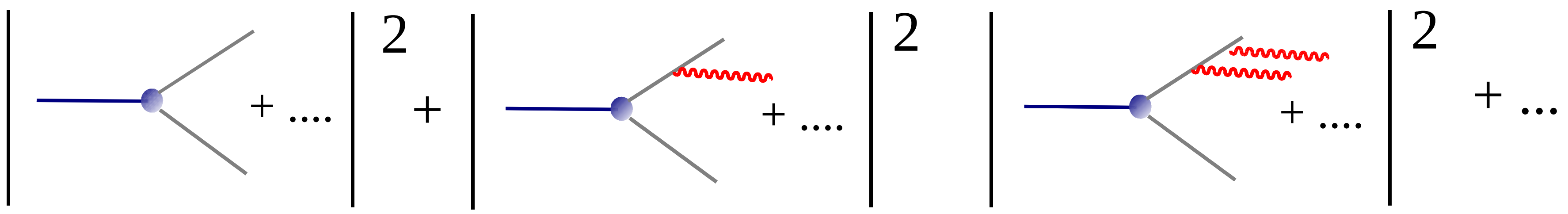
$$\Gamma\,[E^{cut}] = \Gamma_0\, G(E^{cut})$$



Finite terms and UV virtual corrections subtracted
following the prescription by Cirigliano *et al.* '00

Soft-photon resummation

The leading-log series due to multiple photon emission can be summed:



$$\Gamma[\text{up to } n\gamma \text{ with } E<E^{cut}] = \Gamma_0[1 + b\ln(2E^{cut}/M) + b^2\ln^2(2E^{cut}/M)/2 + \ldots + b^n\ln^n(2E^{cut}/M)/n! + \textit{regular terms}]$$

➡ $$\Gamma^{tot}[E] = \Gamma_0 (2E/M)^b [1 + \textit{const} + O(E)]$$

➡ $$\frac{d\Gamma}{dE} = b\,\frac{\Gamma_0}{E}\left(\frac{2E}{M}\right)^b [1 + c + O(E)]$$

This distribution can be integrated down to E → 0 without IR cut-off

$$0 < b = O(\alpha) < 1$$

quite useful for MC

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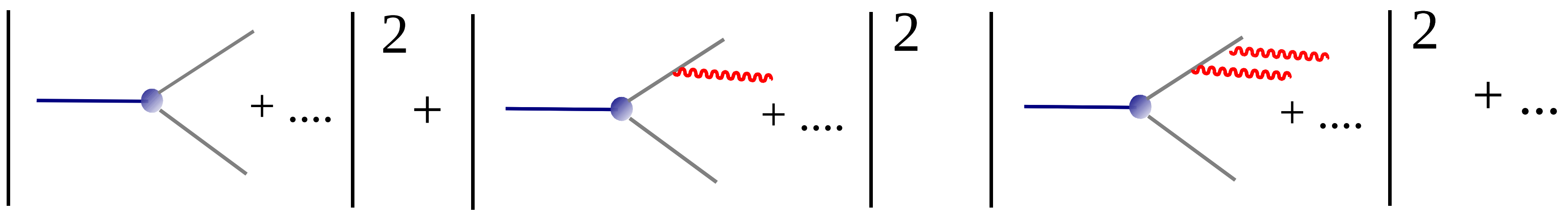

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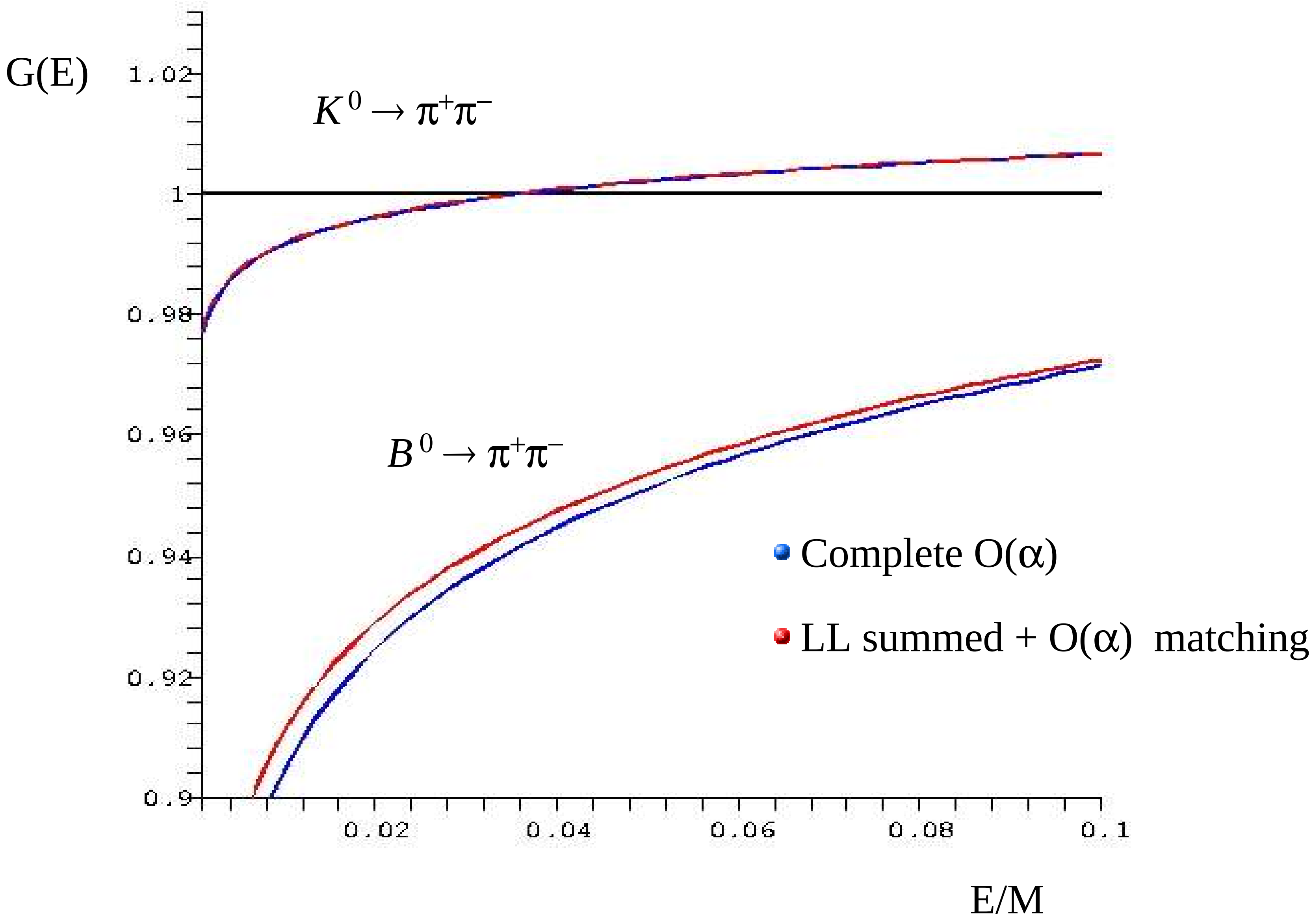
The regular terms must be fixed by a matching condition with the complete $O(\alpha)$ expression

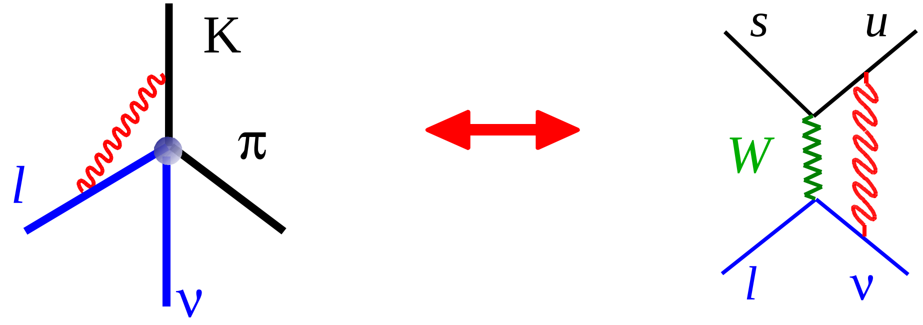
$b, c = O(\alpha)$

$$\Gamma^{tot}[E] = \Gamma_0 G(E)$$



UV structure of the effective theory

Within kaon physics, the largest source of uncertainty arises by the UV structure of virtual e.m. corrections in the effective theory...
[which -in principle- should match the scale dep. of the partonic 4-quark Hamilt.]



..or by the value of the e.m. CT in CHPT

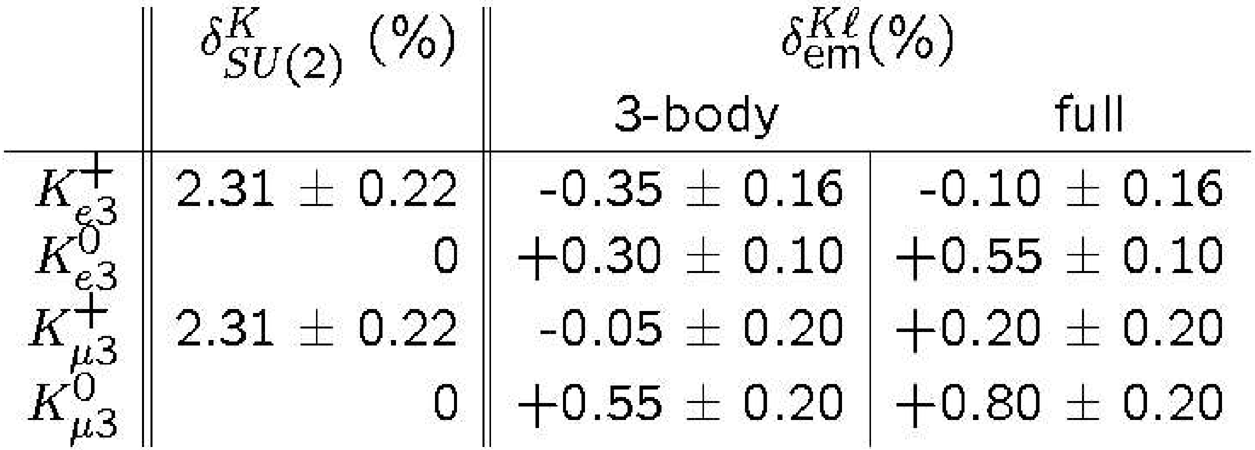
Several progress has been made in the last few year using resonance-saturation and/or large N_c sum rules, but in most cases the problem is still open

Typical size of the error: $(\alpha/\pi)\ \ln(1\ \mathrm{GeV}/M_K) \sim 0.2\ \%$

The K_{l3} case

The master formula for the extraction of V_{us} :

$$\Gamma(K^i_{l3+\gamma}) = C_i \times |V_{us}|^2 \times |f(0)|^2 \times I(\lambda'_+, \lambda''_+, \lambda_0) \times [1 + \delta_{e.m.} + \delta_{SU(2)}]$$



	$\delta^K_{SU(2)}$ (%)	$\delta^{K\ell}_{em}$ (%)	
		3-body	full
K^+_{e3}	2.31 ± 0.22	-0.35 ± 0.16	-0.10 ± 0.16
K^0_{e3}	0	$+0.30 \pm 0.10$	$+0.55 \pm 0.10$
$K^+_{\mu 3}$	2.31 ± 0.22	-0.05 ± 0.20	$+0.20 \pm 0.20$
$K^0_{\mu 3}$	0	$+0.55 \pm 0.20$	$+0.80 \pm 0.20$

th. error ~ 0.3%
[subleading at present stage]

possible consistency checks
with a comparison between
charged & neutral modes

Cirigliano *et al*. '00-'04
Andre '04

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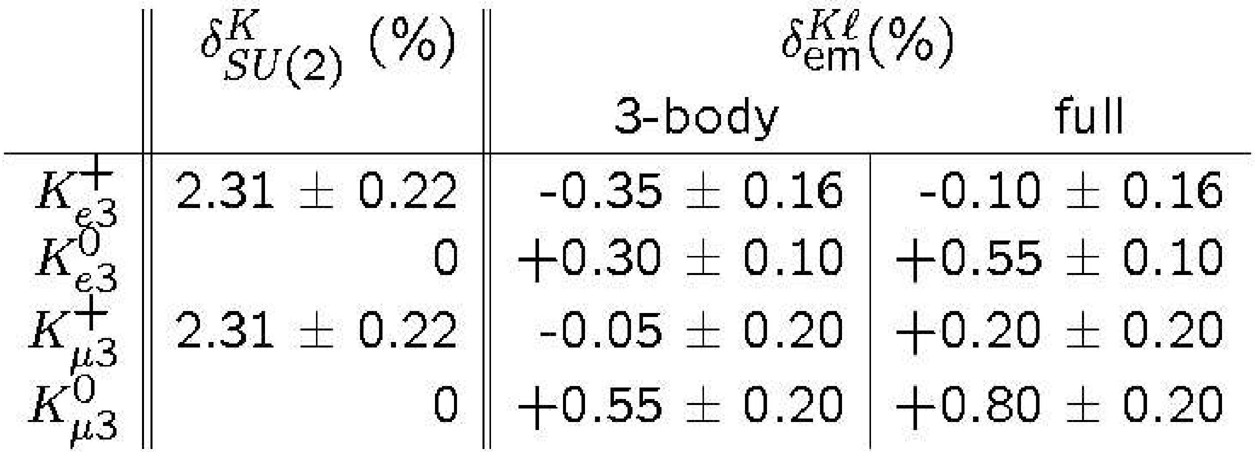

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We are definitely entering an era of high-precision physics in K decays