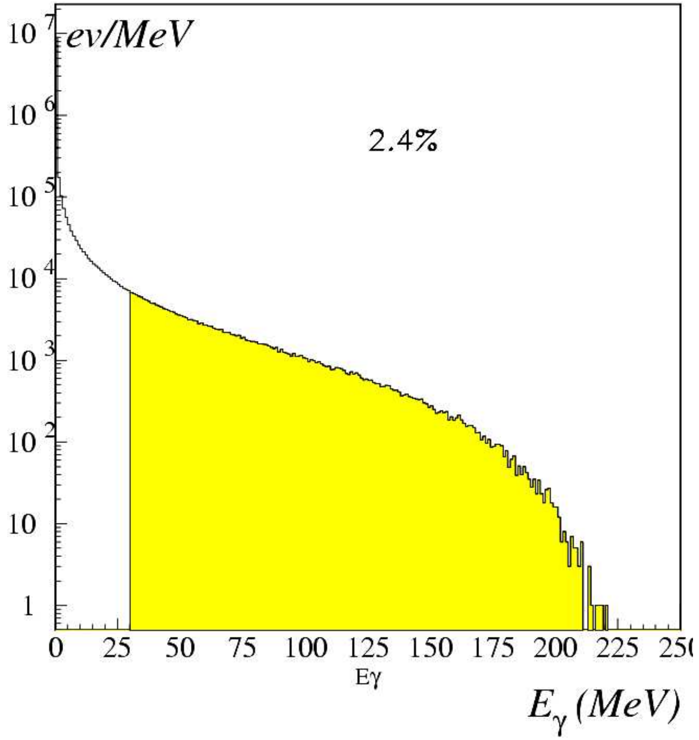
MC generators for radiative kaon decays

Claudio Gatti
Univerista' di Roma 'La Sapienza' – INFN

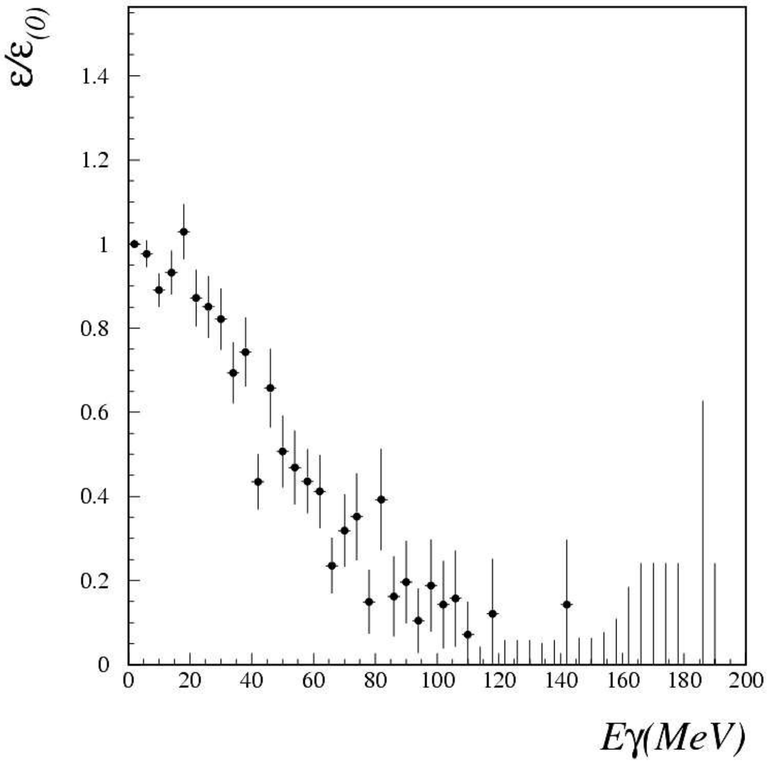
- **Impact of Bremsstrahlung on BR measurement: Acceptance; Event counting.**
- **The KLOE MC generators.**
- **Comparisons.**
- **Conclusion.**

Impact on BR measurement: Acceptance

In a large fraction of Ke3 decays photons have more than 5-10% of CM energy:Softer momentum spectra of π and e;Different acceptance.



$K\rightarrow\pi e\nu\gamma$ photon spectrum (MC)

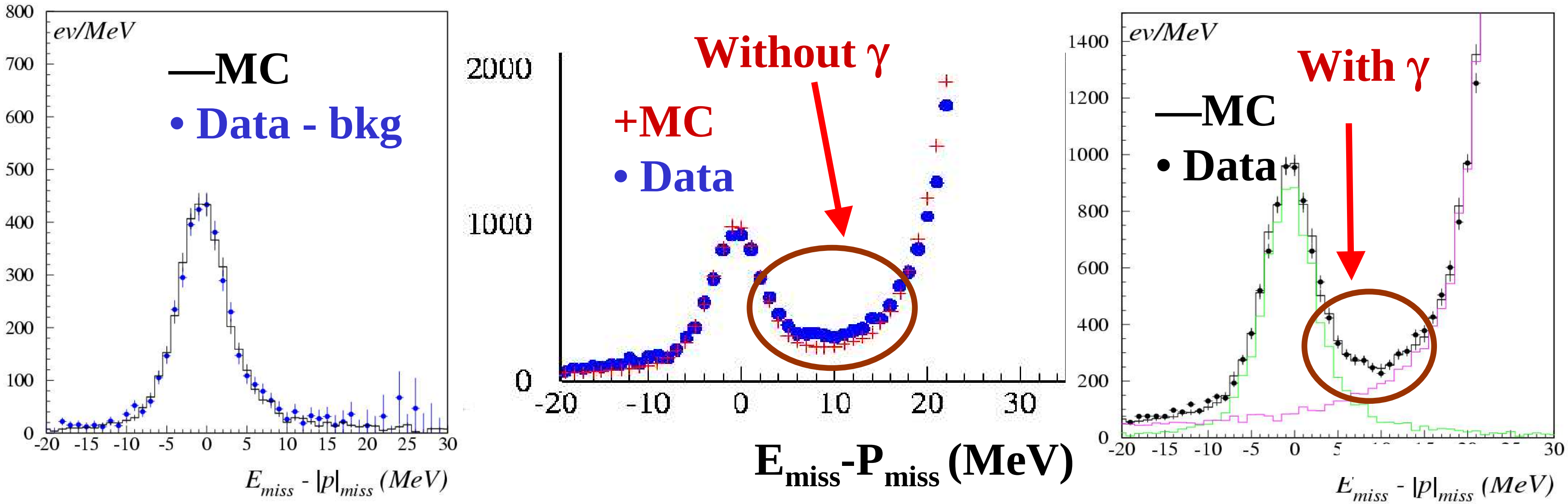


$K_S\rightarrow\pi e\nu\gamma$ acceptance in KLOE (MC)

C.Gatti **Workshop on radiative corrections San Diego March 2005**

Impact on BR measurement: Event counting

Event counting often performed by fitting data distributions.

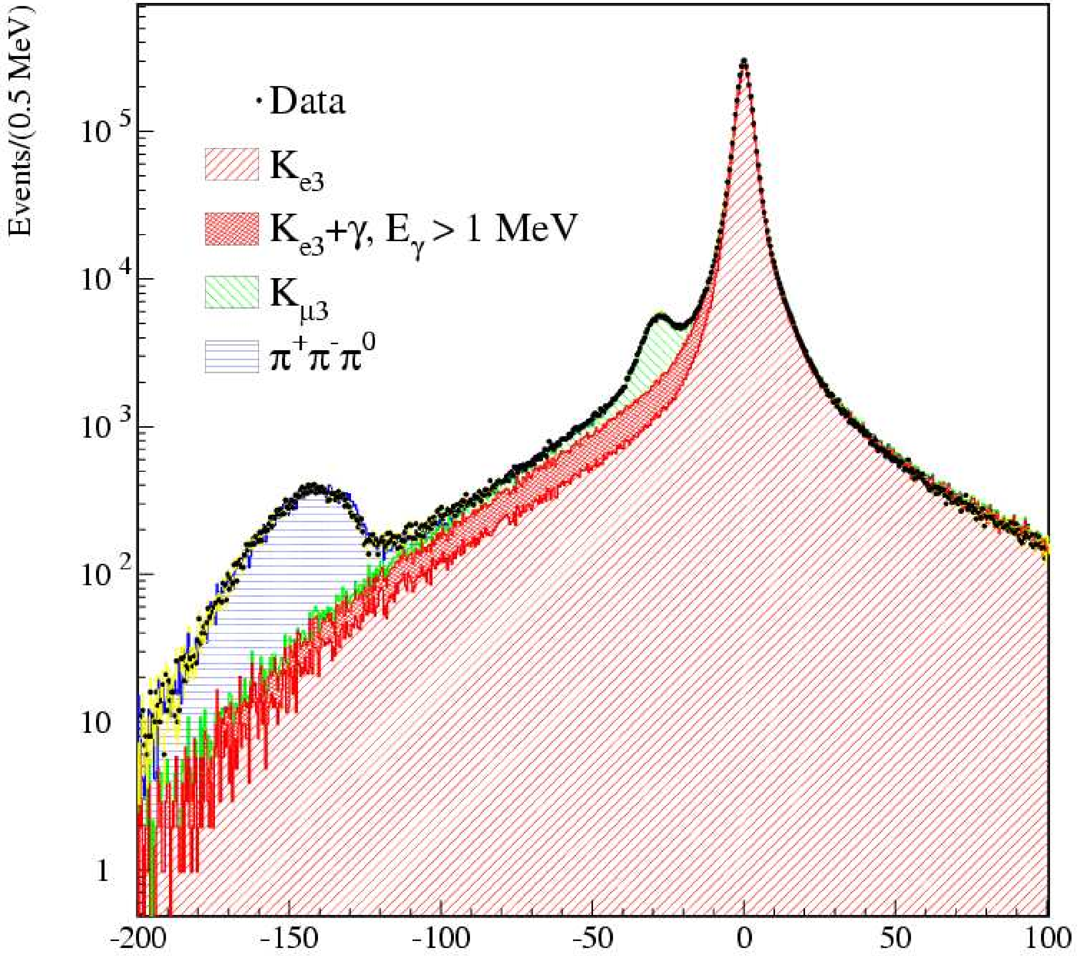


Exemple 1:

In $K_S \rightarrow \pi e \nu$ decays we fit the E_{miss}-P_{miss} distribution. The right tail of the distribution is longer because of the presence of the photons. Neglecting the radiation the event counting is wrong by 1-2%.

Impact on BR measurement: Event counting

Exaple 2:
The K_L BR's are measured by fitting the P_{miss}-E_{miss} distribution. Neglecting the radiative tail the Ke3 BR changes by ~2%.



P_{miss} –$E_{miss}(\pi e)$ distribution in a Ke3-enriched sample

The KLOE MC generators

KLOE studies with high precision decays of both charged and neutral kaons. Therefore, we need a complete description of the radiation in our MC simulation: the photon emission from all the charged particles must be added coherently.

There are two main problems for the simulation of radiative K decays:

- The IR divergency of the decay amplitude at $O(\alpha)$.
- A large production of MC events ($\sim 10^9$ K decays) requires a short time for the generation of a single event ($t<1$ ms). A simple ‘Hit or Miss’ procedure is very ineffective or even impossible for differential decay width that are strongly peaked or divergent in some region of the phase space.

The KLOE MC generators

$$\frac{d\Gamma_{Low}(o(\alpha))}{dE_\gamma} = \frac{\Gamma_0 b}{E_\gamma}$$

The Bremsstrahlung amplitude from Low Theorem.
Γ_0 is the decay width for the bare process.

$$b = -\frac{1}{8\pi}\sum_{mn}\eta_m\eta_n e_m e_n \beta_{mn}^{-1}\ln\frac{1+\beta_{mn}}{1-\beta_{mn}}$$

Solid angle integral (bond factor).

$$\Gamma_{TOT} = \Gamma_0\left(\frac{E_\gamma}{M}\right)^b$$

Adding, in the soft photon limit, the real and virtual contributions to all order in α, the decay width is finite. M is a cut-off energy (the decaying particle mass).

S.Weinberg, Phys.Rev. 140 (1965) 516

Deriving w.r.t. the photon energy:

$$\frac{d\Gamma_{TOT}}{dE_\gamma} = \Gamma_0 b\frac{E_\gamma^{b-1}}{M^b} = \frac{d\Gamma_{Low}(o(\alpha))}{dE_\gamma}\left(\frac{E_\gamma}{M}\right)^b$$

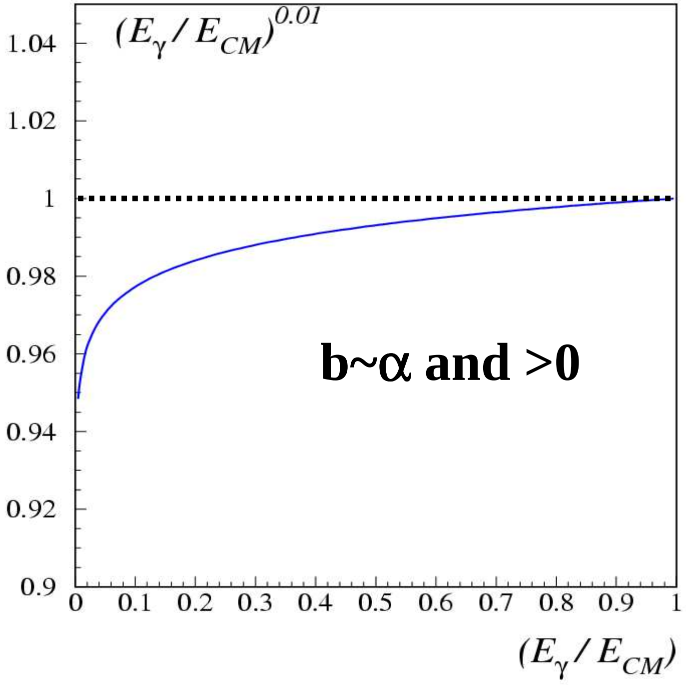
The KLOE MC generators

The recipe:

Use full O(α) amplitude instead of the Low amplitude:

$$\Gamma_{Low}(o(\alpha)) \Rightarrow \Gamma_{Brem}(o(\alpha))$$

$$\Rightarrow \frac{d\Gamma_{TOT}}{dE_\gamma} = \frac{d\Gamma_{Brem}(o(\alpha))}{dE_\gamma}\left(\frac{E_\gamma}{M}\right)^b$$



b~α and >0

Γ_{Brem}(α) mainly from o(p^2) Bijnens et al.
In the Dafne Handbook

The KLOE MC generators

$$|A|^2 = \frac{1}{E_\gamma^{1-b}} \frac{1}{E_l - p_l \cos\theta} \times F(E_\gamma, p_l, \cos\theta)$$

Divergent terms **Flat distribution**

1) Photon energy

$$\left(\frac{M}{E}\right)^{1-b} \Rightarrow F = \int dE \Rightarrow E = E_{\max} \cdot F^{1/b} \quad \mathbf{(0 \leq F \leq 1)}$$

2) Collinearity

$$\frac{1}{(p+q)^2 - m^2} = \frac{1}{2p \cdot q} = \frac{1}{2q(E - p\cos\theta)}$$

Relevant for m = $\mathbf{m_e}$

- **Divergent terms are extracted using the inverse-transform method**
- **A ’Hit-or-Miss’ on the remaning ‘flat’ part is used to weight the kinematical variables extracted from the previous step.**

Comparisons

θ/E(MeV)	10	20	30	40
0°	4.90	3.25	2.36	1.78
10°	2.44	1.65	1.22	0.93
20°	1.85	1.26	0.93	0.71
30°	1.51	1.02	0.76	0.58
40°	1.26	0.86	0.63	0.49
50°	1.06	0.72	0.54	0.41

θ/E(MeV)	10	30
0°	4.93	2.36
20°	1.89	0.956

T. Andre hep/ph 0406006

$$R=BR(Ke3\gamma\ E,\theta)/BR(ke3\gamma)\times 10^{2}$$

θ/E(MeV)	10	20	30	40
0°	4.99	3.28	2.37	1.79
10°	2.51	1.69	1.25	0.96
20°	1.92	1.30	0.96	0.74
30°	1.57	1.06	0.79	0.61
40°	1.31	0.89	0.67	0.51
50°	1.11	0.76	0.57	0.44

M.G. Doncel, Phys.Lett. 32B (1970) 623

Conclusion

- MC generators for neutral and charged kaons into leptonic, semileptonic and into two and three pions have been written and inserted into the KLOE official library.
- The sampling time for an event is <1 ms/evt (except for $k^+ \to e\nu$ t~1.5 ms).
- MC generators use full $O(\alpha)$ amplitude, including also interference terms.
- the extension of the soft-photon approximation to the whole energy range gives a satisfactory description for the kaon radiative decays.
- Absolute difference between MC and order α calculations are $< \sim 10^{-3} \div 10^{-4}$.

C.Gatti 'MC generators for kaon radiative decays'

Kloe note 194 www.lnf.infn.it/kloe/pub/knote/kn194.ps