# Discussion on radiative effects in inclusive s.l. B decays

Paolo Gambino
INFN Torino

# Semileptonic B decays

Short distance electroweak physics at $O(M_W)$: after decoupling heavy dof,

$$\text{Amplitude} = \Sigma_i\ C_i(m_b) \langle O_i(m_b) \rangle.$$

where dim $O_i \geq 6$. $C_i$ evolve from $M_W$ to $m_b$ (large logs), but the dominant (Fermi) operator $J_l J_h$ evolves ONLY due to QED

The logs

$$1+\frac{2\alpha}{\pi}\ln\frac{M_Z}{m_b}$$

Sirlin

are easily resummed. $C_{JJ}(M_W)$ receives very small $O(\alpha)$ corrections.

if Gmu is used as normalization

Haisch,PG

At $m_b$ scale the challenge is to compute $\langle O_{JJ} \rangle$. $m_b \gg \Lambda_{QCD}$ allows HQE.

Only EW effects are QED corrections, real and virtual

# State of the art

$$\frac{d^2\Gamma}{dE_l\, dq^2\, dq_0}$$ can be expressed as double series in $\alpha_s$ and $\Lambda_{QCD}/m_b$ (OPE) with parton model as leading term **No $1/m_b$ correction**

$m_b, m_c$ $\mu_G^2, \mu_\pi^2$ $\lambda_1, \lambda_2$ $\rho_D^3, \rho_{LS}^3$ $\rho_1, \rho_2$

Gremm, Kapustin...

$$\Gamma_{cl\nu} = \frac{G_F^2 m_b^5}{192\pi^3} |V_{cb}|^2 A_{ew} z_0(r) \left(1 + a_1(r)\frac{\mu_\pi^2}{m_b^2} + a_2(r)\frac{\mu_G^2}{m_b^2} + a_3(r)\frac{\rho_D^3}{m_b^3} + a_4(r)\frac{\rho_{LS}^3}{m_b^3}\right)$$

Recent implementation for moments of lept and hadronic spectra including a cut on the lepton energy Bauer et al., Uraltsev & PG

Perturbative QCD Corrections: full $O(\alpha_s)$ and $O(\beta_0\, \alpha_s^2)$ available For the triple differential rate Trott, Uraltsev, Aquila, PG, Ridolfi

NB This assumes that all QED effects in the matrix elements are SUBTRACTED by experiments

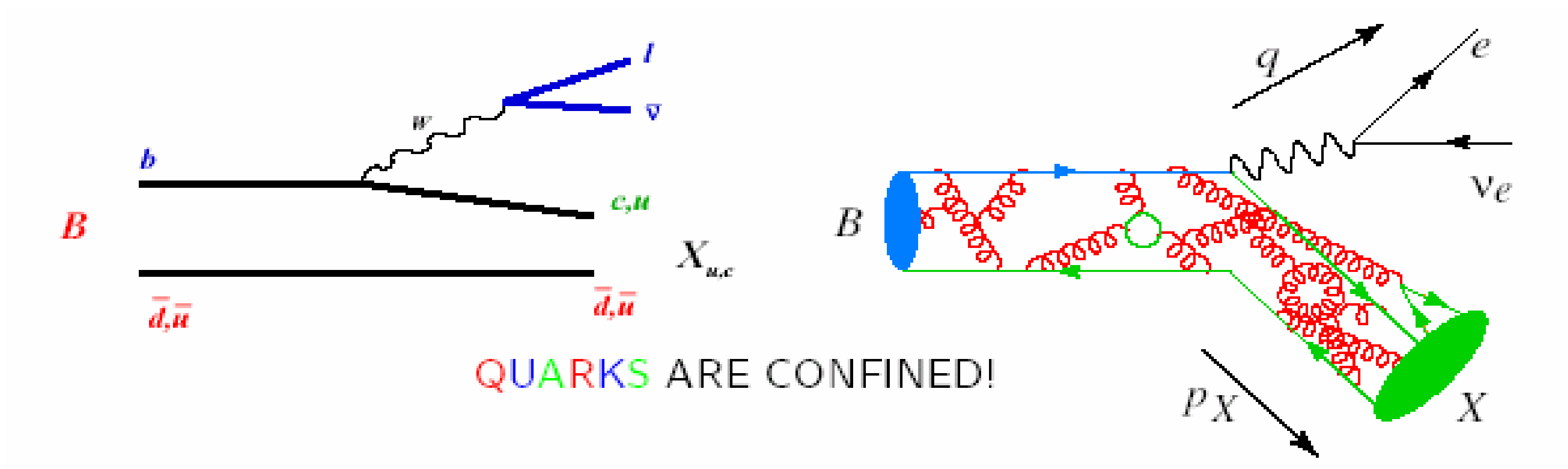
# QED effects

- Can be included in the OPE calculations if the exp is completely inclusive in photons. Can it be done?
- Can be significant close to end points (soft and collinear), especially for electrons. How large can the effect be in first few moments?
- Additional Coulomb $1+\alpha\pi$ for $B^0$ Atwood-Marciano
- Is it better to subtract rad effects at exp level? (remember LEP...)
- Structure effects relevant or parton level sufficient for inclusive decays?

# The advantage of being inclusive

$\Lambda_{QCD} \ll m_b$ : inclusive decays admit systematic expansion in $\Lambda_{QCD}/m_b$
Non-pert corrections are generally small and can be controlled

Hadronization probability =1 because we sum over all states
Approximately insensitive to details of meson structure as $\Lambda_{QCD} \ll m_b$
(as long as one is far from perturbative singularities)




$$\frac{d^2\Gamma}{dE_l\, dq^2\, dq_0}$$ can be expressed as double series in $\alpha_s$ and $\Lambda_{QCD}/m_b$ (OPE)
with parton model as leading term **No $1/m_b$ correction!**

# A double expansion

$\frac{d^2\Gamma}{dE_l dq^2 dq_0}$ can be expressed in terms of *structure functions* related to Im of

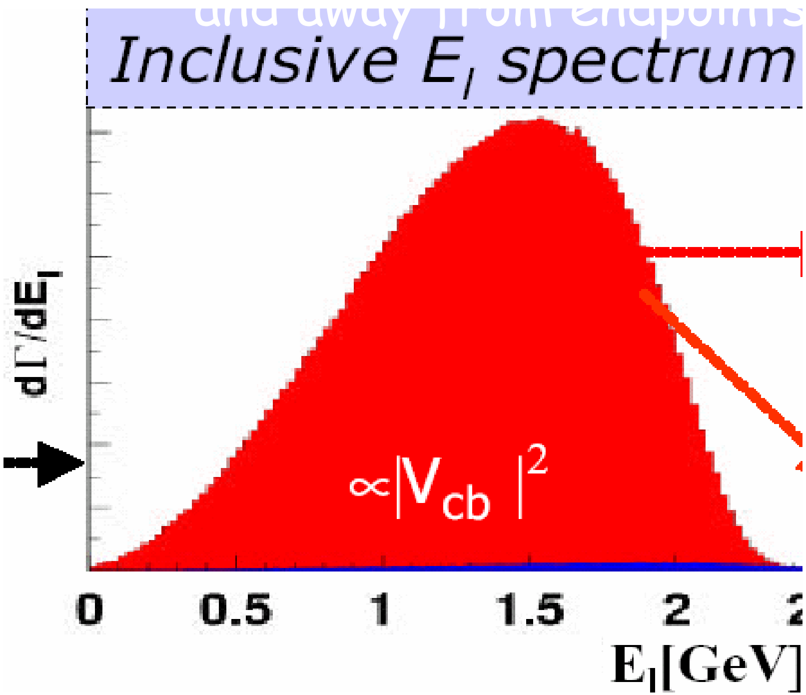
$$h_{\mu\nu}(q^2, q_0) = \frac{1}{2M_B} \langle B | \int \mathrm{d}^4x \, \mathrm{e}^{-iqx} \, iT \left\{ J_\mu(x), J_\nu^\dagger(0) \right\} | B \rangle$$

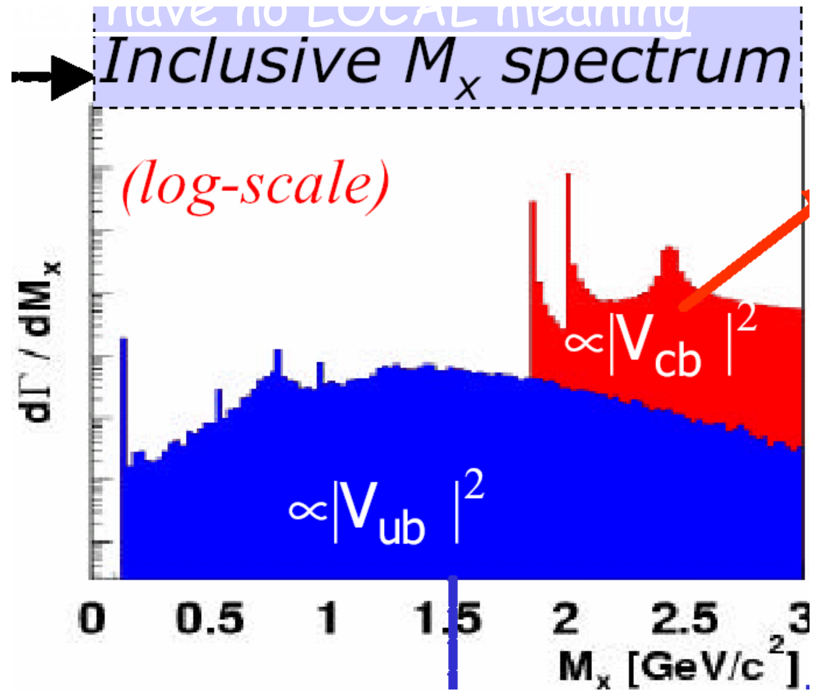
**OPE (HQE):** $T\, J(x)\, J(0) \approx c_1 \bar{b}b + c_2 \bar{b} \vec{D}^2 b + c_3 \bar{b}\sigma \cdot G b + \ldots$

- The leading term is parton model, $c_i$ are series in $\alpha_s$
- New operators have non-vanishing expection values in B and are suppressed by powers of the energy released, **$E_r \sim m_b - m_c$**
- **No $1/m_b$ correction!**

OPE predictions can be compared to exp only after SMEARING and away from endpoints: they have no LOCAL meaning

# Leptonic and hadronic spectra




Total **rate** gives CKM elmnts; **shape** tells us about B structure

# Using moments to extract HQE parameters

We do know something on HQE par.
need to check consistency.

- $M_{B*}$-$M_B$ fix $\mu_G^2$= 0.35±0.03
- Sum rules: $\mu_G^2 < \mu_\pi^2$, $\rho_D^3 > -\rho^3_{LS}$...

Central moments can be VERY sensitive to HQE parameters

$$\left\langle \left(M_X^2 - \langle M_X^2 \rangle\right)^2 \right\rangle \approx \left[1.3 + 0.4(m_b - 4.6) - (m_c - 1.2) + 5(\mu_\pi^2 - 0.4) - 6(\rho_D^3 - 0.1) + \ldots\right] GeV^4$$

Variance of mass distribution

BUT: OPE accuracy deteriorates for higher moments (getting sensitive to local effects)

Provided cut is not too severe (~1.3GeV) the cut moments give additional info

# Fit Results

kinetic mass scheme

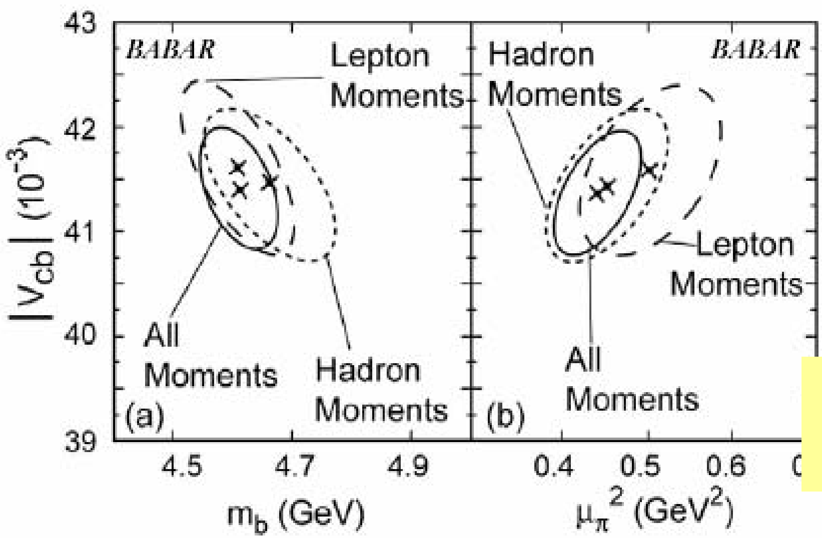
$$|V_{cb}| = (41.4 \pm 0.4_{exp} \pm 0.4_{HQE} \pm 0.2_{\alpha_s} \pm 0.6_{\Gamma_{SL}}) \times 10^{-3}$$
$$Br(B \to X_c e \nu) = (10.61 \pm 0.16_{exp} \pm 0.06_{HQE})\%$$
$$m_b(1\ GeV) = (4.61 \pm 0.05_{exp} \pm 0.04_{HQE} \pm 0.02_{\alpha_s})GeV$$
$$m_c(1\ GeV) = (1.18 \pm 0.07_{exp} \pm 0.06_{HQE} \pm 0.02_{\alpha_s})GeV$$

$$\mu_\pi^2 = (0.45 \pm 0.04_{exp} \pm 0.04_{HQE} \pm 0.01_{\alpha_s})GeV^2$$
$$\mu_G^2 = (0.27 \pm 0.06_{exp} \pm 0.03_{HQE} \pm 0.02_{\alpha_s})GeV^2$$
$$\rho_D^3 = (0.20 \pm 0.02_{exp} \pm 0.02_{HQE} \pm 0.00_{\alpha_s})GeV^3$$
$$\rho_{LS}^3 = (-0.09 \pm 0.04_{exp} \pm 0.07_{HQE} \pm 0.01_{\alpha_s})GeV^3$$

Kinetic scheme:
Small pert corrections
Minimal set of parmts
No $1/m_c$ expansion
Uraltsev & PG

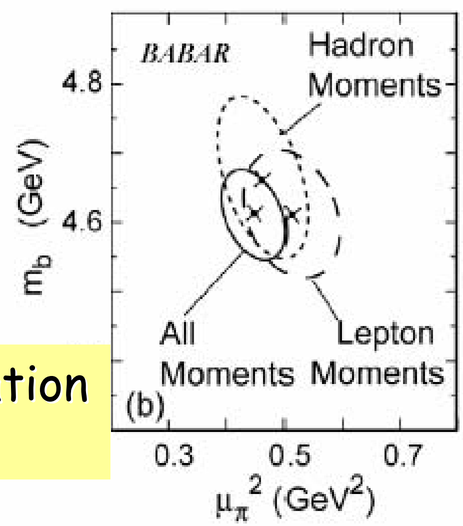
**Strong correlation between $m_b$ and $m_c$:**

$m_b(1\ GeV) - m_c(1\ GeV) =$
$(3.44 \pm 0.03_{exp} \pm 0.02_{HQE} \pm 0.01_{\alpha_s})$ GeV




**2D projections of the fit result:**

$\Delta\chi^2 = 1$ ellipses




No sign of deterioration for higher cuts