SUBLEADING SHAPE FUNCTIONS IN INCLUSIVE B DECAYS

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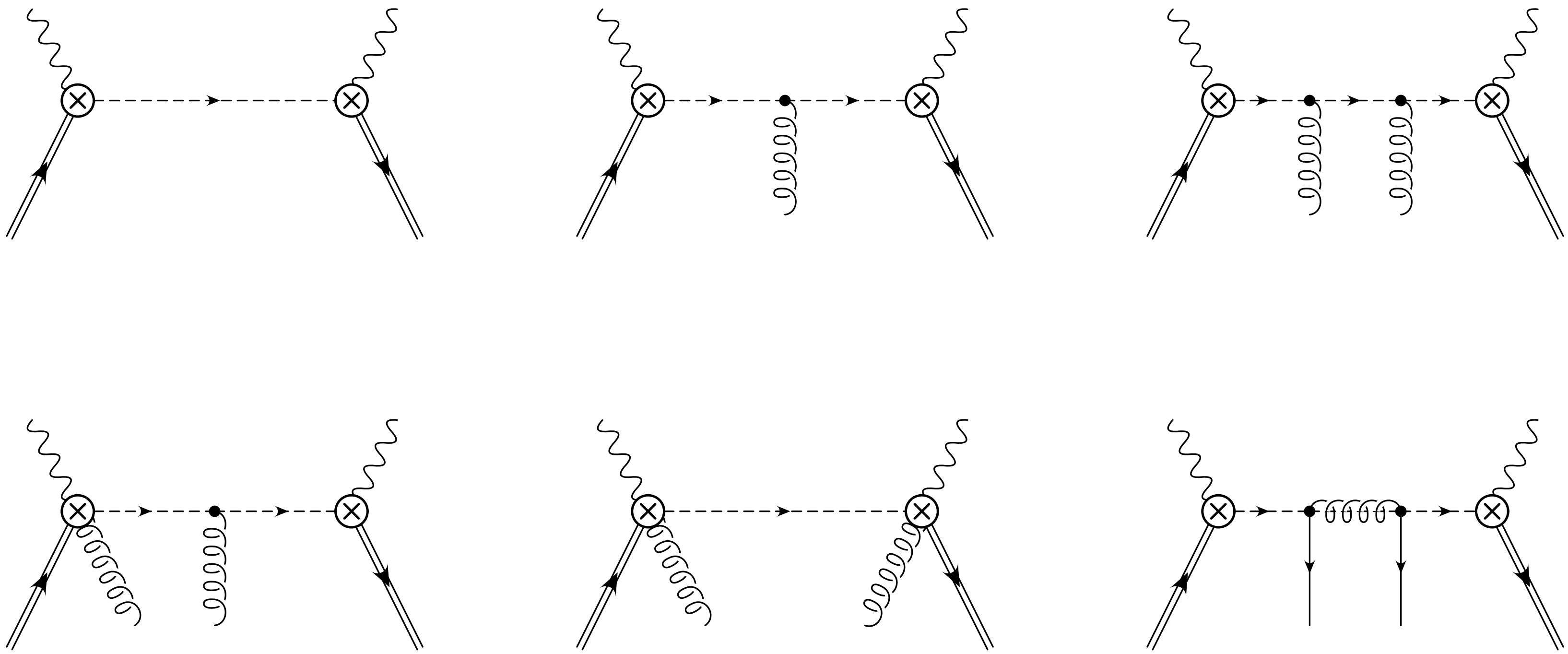
Motivation

- Goal: Calculate $\frac{1}{m_b}$ corrections to $\bar{B} \to X_u\, l^- \bar{\nu}, \bar{B} \to X_s \gamma$
- Method: SCET $\Rightarrow$ Systematic expansion in $\sqrt{\lambda} \equiv \sqrt{\Lambda/m_b}$
- Light cone coordinates:
  - $n, \bar{n} = (1, 0, 0, \pm 1)$
  - $a^\mu = (n \cdot a) \frac{\bar{n}^\mu}{2} + (\bar{n} \cdot a) \frac{n^\mu}{2} + a_\perp^\mu$
- Modes in SCET:
  - Hard Collinear: $(n \cdot p) \sim \Lambda$, $(\bar{n} \cdot p) \sim m_b$, $p_\perp \sim \sqrt{m_b \Lambda}$
  - Soft: $(n \cdot p) \sim \Lambda$, $(\bar{n} \cdot p) \sim \Lambda$, $p_\perp \sim \Lambda$
- $d\Gamma \sim H \cdot J \otimes S$ at LO in λ
- What happens at NLO ?

SCET at Subleading Power

- $d\Gamma \sim W^{\mu\nu} L_{\mu\nu}$
$$W^{\mu\nu} \sim \langle \mathrm{T}\left\{J^{\dagger\mu}(0), J^{\nu}(x)\right\}\rangle$$
- $\frac{1}{m_b}$ corrections $\rightarrow J, \mathcal{L}$ at $\mathcal{O}(\sqrt{\lambda}^{\,2})$
- $J = \bar{q}\Gamma b \quad \rightarrow \quad J^{(0)} = \bar{\mathfrak{X}}\Gamma\mathcal{H}$
  $\rightarrow \quad J^{(1)} = \cdots$
  $\rightarrow \quad J^{(2)} = \cdots$
- $\mathcal{L} = \bar{\psi} i \not{D} \psi \quad \rightarrow \quad \mathcal{L}_{\xi}^{(0)}, \mathcal{L}_{\xi}^{(1)}, \mathcal{L}_{\xi}^{(2)}$
  collinear-collinear interaction
  $\rightarrow \quad \mathcal{L}_{\xi q}^{(1)}$
  soft-collinear interaction
  $\rightarrow \quad \mathcal{L}_{HQET}^{(2)}$
  heavy quark interaction

Hadronic Tensor



- $W^{(1)}_{\mu\nu} = 0$ (Can choose $p_\perp = v_\perp = 0$)
- Factorization at NLO $H \cdot J_i \otimes s_k$
- Bi, Tri, Quadri local operators $\Rightarrow s_k(\omega, \ldots)$
- At tree level only $s_k(\omega)$

Subleading Shape Functions at Tree Level

- LO: $\int d\omega\, e^{-\frac{i}{2}\omega\bar{n}\cdot x}\, S(\omega) = \langle \left(\bar{h}S\right)_0 \left(S^\dagger h\right)_{x_-}\rangle$
- "HQET": $\frac{1}{m_b}\int d\omega\, e^{-\frac{i}{2}\omega\bar{n}\cdot x}\, s(\omega) = \langle i\int d^4z\, T\{\left(\bar{h}S\right)_0 \left(S^\dagger h\right)_{x_-}\, \mathcal{L}_h^{(2)}(z)\}\rangle$
- "Chromo-electro-magnetic": $\int d\omega\, e^{-\frac{i}{2}\omega\bar{n}\cdot x}\, t(\omega) =$
$$-\int_0^{\bar{n}\cdot x/2} dt\, \langle \bar{h}(0)\, \frac{\not{\bar{n}}}{2}\, [0,tn]\, \gamma_\perp^\mu n^\nu\, gG_{\mu\nu}(tn)\, [tn,x_-]\, h(x_-)\rangle$$
- "Kinetic":
$$\int d\omega\, e^{-\frac{i}{2}\omega\bar{n}\cdot x}\, u(\omega) = -i\int_0^{\bar{n}\cdot x/2} dt\, \langle \bar{h}(0)\, [0,tn]\, (iD_\perp)^2(tn)\, [tn,x_-]\, h(x_-)\rangle$$
- "Chromo-magnetic":
$$\int d\omega\, e^{-\frac{i}{2}\omega\bar{n}\cdot x}\, v(\omega) = -i\int_0^{\bar{n}\cdot x/2} dt\, \langle \bar{h}(0)\, \frac{\not{\bar{n}}}{2}\, [0,tn]\, \sigma_{\mu\nu}^\perp\, G_\perp^{\mu\nu}(tn)\, [tn,x_-]\, h(x_-)\rangle$$
- "Four quark" SF

Moments of the Shape Functions

$$W^{\mu\nu} = \int d\omega\, \delta(n\cdot p+\omega)\Big\{\left(n^\mu v^\nu + n^\nu v^\mu - g^{\mu\nu} - i\epsilon^{\mu\nu\alpha\beta}\, n_\alpha v_\beta\right)$$
$$\times\left[\left(1+\frac{\omega}{m_b}\right)S(\omega) + \frac{s(\omega)+t(\omega)}{m_b} + \frac{u(\omega)-v(\omega)}{\bar n\cdot p}\right]$$
$$-\,2(n^\mu v^\nu + n^\nu v^\mu)\,\frac{t(\omega)}{\bar n\cdot p} + 2n^\mu n^\nu\left[-\frac{\omega\, S(\omega)}{m_b} - \frac{t(\omega)}{m_b} + \frac{t(\omega)+v(\omega)}{\bar n\cdot p}\right]\Big\}$$

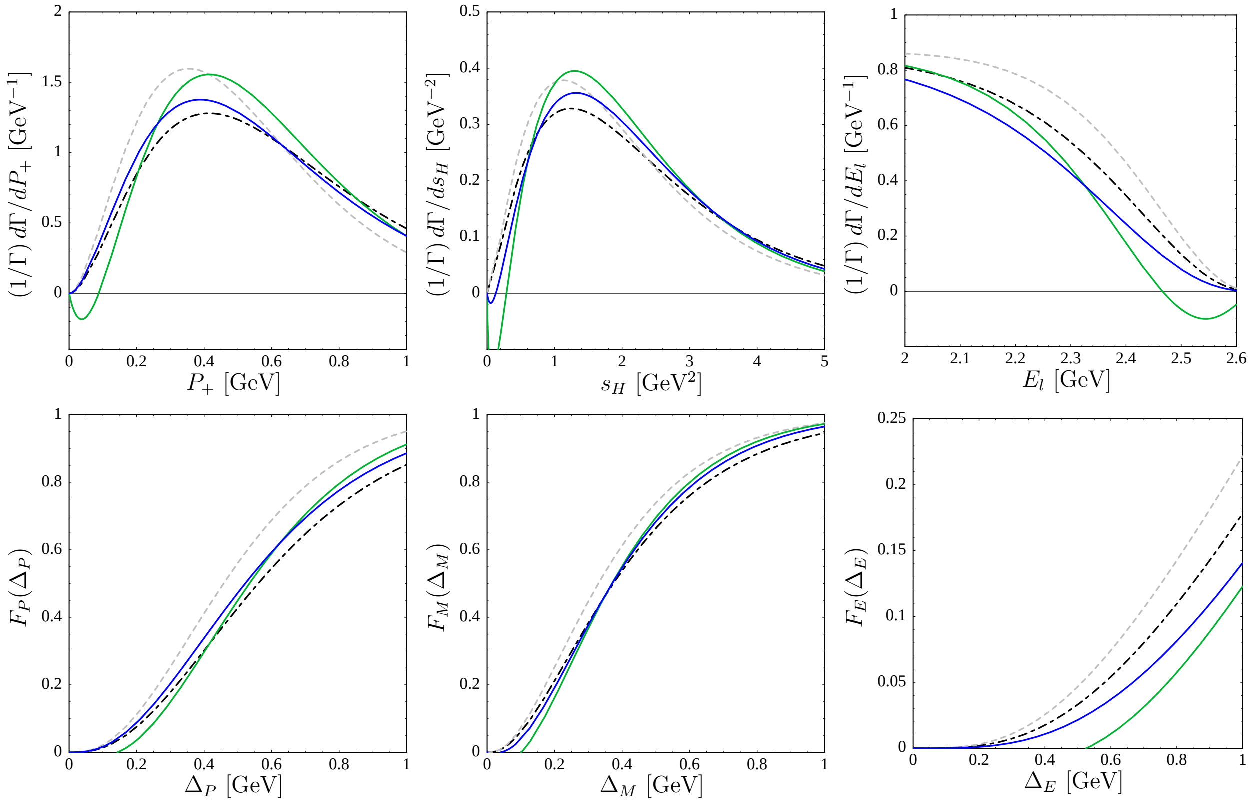
- $s(\omega)$ absorbed into $S(\omega)$
- SF's moments $\Leftrightarrow$ HQET parameters
- $\int d\omega\ \{s(\omega), t(\omega), u(\omega), v(\omega)\} = 0$

$$\int d\omega\,\omega\, s(\omega) = -\frac{\lambda_1+3\lambda_2}{2} \qquad \int d\omega\,\omega\, u(\omega) = \frac{2\lambda_1}{3}$$
$$\int d\omega\,\omega\, t(\omega) = -\lambda_2 \qquad \int d\omega\,\omega\, v(\omega) = \lambda_2$$

Four Quark Operators

- At $\mathcal{O}(g^2)$ tree level: "4-quark" light cone operator: $O_5(x_-) =$
$$\frac{\pi\alpha_s}{\bar{n}\cdot p} \int_0^{\bar{n}\cdot x/2} dt_1 \int_{t_1}^{\bar{n}\cdot x/2} dt_2 \left(\bar{h}S\right)_0 \Gamma_i \not{n}\gamma_\rho^\perp t_a \left(S^\dagger q\right)_{t_1 n} (\bar{q}S)_{t_2 n} \gamma_\perp^\rho \not{n} \Gamma_j t_a \left(S^\dagger h\right)_{x_-}$$
- $O_5 \Rightarrow f_u, f_v$. How big are they?
- zero, first moments vanish
- O_5 vanishes at vacuum-insertion approximation
- – Lee, Stewart (hep-ph/0409045) - effect on E_l could be 180%
  – Neubert (hep-ph/041027) - effect is $\epsilon \cdot 20\%$ for E_l and $\epsilon \cdot 3\%$ for P_+, $\epsilon \sim 0.1$. Correction vanish for $q \neq$ spectator
  – Beneke, Campanario, Mannel, Pecjak (hep-ph 0411395) - effect is $\sim 5\%$. Correction vanish for $q \neq$ spectator
- $f_u, f_v \Rightarrow$ isospin breaking

Subleading SF's Effect on Spectra



- Dash-dotted: Leading order
- Dashed: Kinematic Correction
- Model I
- Model II

Comparison with Other Studies

- QCD $\rightarrow$ HQET
  1. Bauer, Luke, Mannel (hep-ph/0102089): $W^{\mu\nu}$, photon spectrum for $\bar{B} \rightarrow X_s \gamma$
  2. Bauer, Luke, Mannel (hep-ph/0205150): lepton spectrum for $\bar{B} \rightarrow X_u \, l^- \bar{\nu}$
  3. Burrel, Luke, Williamson (hep-ph/03122366): s_H spectrum for $\bar{B} \rightarrow X_u \, l^- \bar{\nu}$
- QCD $\rightarrow$ SCET$\rightarrow$ HQET
  4. Lee, Stewart (hep-ph/0409045)
  5. Bosch, Neubert, P. (hep-ph/0409115)
  6. Beneke, Campanario, Mannel, Pecjak (hep-ph/0411395)
- $1 \neq 2$, $1 \neq 3$
- $4 \neq 1, 2$ $5 \neq 1, 2$ $6 \neq 1, 2$
- $3 \in 5$, $3 \in 6$
- $6 \sim 5$

Conclusion

- SCET $\Rightarrow$ subleading SF's
- At tree level: $t(\omega)$, $u(\omega)$, $v(\omega)$
- 4-quark operators
- Power corrections are small
- Disagreement with previous studies