$V_{ub}$ from Lattice QCD
Exclusive Semileptonic Decays

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Quenched calculations of $B \to \pi l \nu$ decay are in good shape.

![Graph](image)

Figure 1. $B \to \pi l \nu$ form factors by different lattice groups.


Issues:

- Light fermion methods.
- Chiral extrapolations.
- $q^2$ dependence of lattice errors.
- Stable vs. unstable mesons.
Three types of light unquenched fermions:

**Naive/staggered**
Fastest. Nice chiral behavior, but fermion doubling requires taking root of determinant.

**Wilson**
10-100 times slower. “Exceptional configurations”: poorer statistics/configuration, more steps to an independent configuration. Messy chiral breaking correction operators.

**DW/overlap/GW**
Very pretty behavior, but a factor of 10-100 slower still. Need 25-50 sites in the fifth dimension.

With unquenched improved staggered fermions, sufficiently simple quantities are beginning to line up well with experiment.

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**FIG. 1:** LQCD results divided by experimental results for nine different quantities, without and with quark vacuum polarization (left and right panels, respectively). The top three results are from our $a = 1/11$ and $1/8$ fm simulations; all others are from $a = 1/8$ fm simulations.
What are “sufficiently simple quantities”?

Stable mesons (not too close to thresholds) are the Golden Quantities of lattice QCD.

- No final state effects.
- Lightest in flavor channel → best statistical errors.
- Small, good volume errors.

Many of the most important quantities for determining standard model parameters are in this category.

- $\pi, K, \psi, \Upsilon \rightarrow m_q$
- $K\bar{K}$ mixing
- $B\bar{B}$ mixing
- $B$ semileptonic decay
- $D$ leptonic and semileptonic decay
The role of charm decays.

- $f_D/D \to \pi l\nu$ and $f_{D_s}/D \to K l\nu$ give 2% tests of the amplitudes similar to the ones required for analysis of the $B$ physics of the unitarity triangle.

- CLEO-c $f_d$ and $f_{D_s}$ plus gold-plated lattice calculations give
  - $V_{cd}$ to 2% (currently 7%).
    (New 2% test of CKM unitarity.)
  - $V_{cs}$ to 2% (currently 12%).
  - Improve $B$ physics with $D$ physics: CLEO-c $F_D$ plus lattice $f_B/f_D$ give $f_B$ to 2%.
After unquenching, (one of?) toughest remaining errors is likely to come from chiral extrapolation. Good understanding of the chiral logs will be required, particularly when extrapolating from large $m_l$.

Kronfeld and Ryan:
Even for quantities like $\xi_f \equiv f_{B_s}/f_B$, for which most lattice errors cancel, uncertainties in the chiral parameters lead to a 10% uncertainty when extrapolating down from $m_l \sim m_s/2$. 

Figure 1: Plot of the chiral logarithm as the mass ratio $r = m_q^2/m_s$ is varied, compared with a straight line $\xi$ for $0 < r < 1$. The difference between the curve and the $\xi$ is shown in the inset.

Figure 2: Plot of $\xi_f$ against $r$ for several values of the low-energy constant: $f_d(1$ GeV) = 0.2, 0.5, 0.8 GeV$^{-2}$. Also shown is the linear extrapolation with $\xi_f(r_d) = 1.15 \pm 0.05$. 

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Chiral extrapolation of $f_\pi$.

**Wilson fermions.**

Wilson data (JLQCD) ends at $m_s/2$, staggered at $m_s/6$.

**Staggered fermions.**

- Lowest two $f_\pi$ s extrapolate linearly to within 2%.
- $m_{\text{sea}}^{u,d} = \{ m_s/2.3, m_s/4.5 \}$
- $m_{\text{val}}^{u,d}/m_s$
Currently, pion recoil momentum range is limited by statistical and discretization errors.

Proposal to ameliorate: Moving NRQCD (Foley and Lepage).

- Remove $B$ momentum as well as rest energy from $b$ Lagrangian. Quantize small momentum fluctuations around large momentum.

- Work in “spectator $\pi$ Breit” frame: $P = \sqrt{\Lambda M_B}$. Momenta of pion, $u$ quark, and residual momentum of $b$ are $\sim 800$ MeV at maximum recoil.

\[
L = \chi^\dagger \left( iD_t + i\bar{v} \cdot \bar{D} + \frac{1}{2m\gamma} \left( \bar{D}^2 - (\bar{v} \cdot \bar{D})^2 \right) + \ldots \right) \chi
\]
Unquenched staggered $D \to \pi l \nu$ form factors.

In progress. Not blessed, not preliminary, not nothin’, just to say we’re working on them.

$B \to \pi l \nu$ at moderate recoil virtually identical in difficulty.

Figure 1: Real analysis for $D \to \pi$ form factors. The statistics for $am_l = 0.01$ is about $500 \times 4$, whereas it is about 400 for $am_l = 0.02$ and 0.03. Vertical lines are put at $am_l = 0.007$ and 0.005.
Topics for Discussion.

- For $B \rightarrow \pi l \nu$, theorists and experimentalists must work together to understand each others’ uncertainties (unlike the rest of lattice’s golden CKM quantities). Theorists’ and (I presume) experimentalists’ uncertainties will be $q^2$ dependent, and thought is required to find the optimal range from which to extract $V_{ub}$.

  - What is the $q^2$ dependence of the theoretical and experimental uncertainties?

- Vector decays. $B$ semileptonic decays into $\rho l \nu$ and $\omega l \nu$ contain an additional component of theoretical murkiness compared to $B \rightarrow \pi l \nu$. (Multihadron states in Euclidean and Minkowski space are not trivially related.)

  - Suppose perfect theory: how much better a $V_{ub}$ would you get by combining all exclusive $b \rightarrow u$ decay, compared with $B \rightarrow \pi l \nu$ alone?
Summary.

- With unquenched improved staggered sea quarks, quarkonium quantities that were known in advance to be easiest agree with experiment at their predicted levels: 2-3%, or 10-15 MeV.

- Quantities more sensitive to corrections (e.g., hyperfine splittings) are agreeing at the 20 MeV level. We know the sources of the remaining errors, and a straightforward (but timeconsuming) program exists to make them as small as we want.

- Therefore, masses in the $B_c$ system should be predictable at the 20 MeV level.
Recent progress in unquenched staggered fermion phenomenology.

For simple enough quantities, significant deviations from experiment in quenched calculations disappear in unquenched.

- $n_f = 3$, $a = 1/8$ fm and $1/11$ fm.

- Tune $m_u = m_d$, $m_s$, $m_c$, $m_b$, and $\alpha_s$ using $m_\pi$, $m_K$, $m_{D_s}$, $m_{\Upsilon}$, and $\Delta E_{\Upsilon}(2S1S)$.

⇒

- $\alpha_{\overline{MS}} = 0.119(3)$

- $m_s(2 \text{ GeV}) = 80(20)$ MeV

Results from several US and UK collaborations: MILC, HPQCD (Cornell, Glasgow (UKQCD), Fermilab, …)
The role of charm factories in high precision lattice phenomenology.

Many masses are known to 1% accuracy, but few amplitudes (such the ones required for analysis of $B$ physics).

ψ and Υ physics.

Dozens of gold-plated (1%) lattice tests now possible.

• Masses, spin splittings for S, P, D states.

• Leptonic widths for S states.

• Electromagnetic transition form factors for $P \rightarrow S$ states, $S \rightarrow P$ states.
In quarkonium systems, potential models give us a much more quantitative understanding of the dynamics than we have with most hadrons.

We can estimate in advance which correction operators are most important, how big finite volume errors are likely to be, which quantities are insensitive to errors, which are not . . .