Constraints on Isgur-Wise function derivatives

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Caltech
Definitions

\[ \frac{d\Gamma(B \to D^* l \bar{\nu})}{dw} \propto \sqrt{w^2 - 1}|V_{cb}|^2 \mathcal{F}_{D^*}(w)^2 \]

\[ \frac{d\Gamma(B \to D l \bar{\nu})}{dw} \propto (w^2 - 1)^{3/2}|V_{cb}|^2 \mathcal{F}_D(w)^2 \]

\[ w = \nu \cdot \nu' = \frac{m_B^2 + m_{D(*)}^2 - q^2}{2m_B m_{D(*)}} \]

HQET \( \Rightarrow \mathcal{F}_{D(*)}(w) = \xi(w) \) with \( \xi(1) = 1 \)

IW functions for HQET P-wave multiplets: \( \tau_{1/2} \) and \( \tau_{3/2} \) (not normalized)
Extrapolation to $w = 1$

- Rates vanish at $w = 1 \Rightarrow$ Measure and extrapolate down
- Limited data $\Rightarrow$ Need constraints on $\mathcal{F}_D(*)$ shape
  
  \[
  \frac{\mathcal{F}_D(*) (w)}{\mathcal{F}_D(*) (1)} = \left[ 1 - \rho^2_D(*) (w - 1) + \sigma^2_D(*) (w - 1)^2 / 2 + \cdots \right]
  \]
- Linear underestimates $|V_{cb} \mathcal{F}_D(*) (1)|$ by $\sim 3\%$
- Including $\sigma^2_D(*)$ (with constraints) desirable
Linear fit vs. quadratic

\[ B \rightarrow D^* \ell \bar{\nu} \ (\text{CLEO'95}) \]

Caprini, Lellouch, Neubert, NPB 530(1998)153
Dispersive Constraints

Analyticity of 2-pt functions, positivity of spectral functions (Boyd et al., Caprini et al.)

$\Rightarrow$ Constraints on form factors. E.g.,

- $-0.17 < \rho_D^2 < 1.51$
- $\mathcal{F}_D(w)/\mathcal{F}_D(1) \approx 1 + f(\rho_D^2)$
- $\Rightarrow \sigma_D^2 = 2.10\rho_D^2 - 0.30$

Can optimize resultant constraints
HQET Sum Rules

Basic Idea: Duality (OPE) ⇒

\[ \sum \text{Hadronic transitions} = \text{free quark transition (at leading order)} \]

Simplest Approach:

\[ T(q^0) = i \int d^4x e^{-iq \cdot x} \langle B | T \{ J^\dagger(x), J(0) \} | B \rangle \]

with \( J(x) = \bar{c} \nu \cdot \Gamma b \)

\[ \int_0^\infty d\epsilon \ \text{Im}[T(\epsilon)] \sim \sum_{X_c} |\langle X_c | J | B \rangle|^2 \]

\[ \sim \sum_{X_c} \xi_{X_c}(w)^2 \]
More Generally

Consider *nonforward* matrix element:

- \( \langle B(v_f)|T\{\cdots\}|B(v_i)\rangle \)
- \( J_i \) and \( J_f \) with \( J_i = \bar{c}v_i \cdot \Gamma_i b \), etc.
- Define \( \omega_{if} = v_i \cdot v_f \), \( \omega_i = v_i \cdot v' \), \( \omega_f = v_f \cdot v' \)
- \( \xi(\omega_{if})F(\omega) = \sum_{X_c} G(\omega)\xi_{X_c}(\omega_i)\xi_{X_c}(\omega_f) \)

Derivatives wrt velocity variables at zero recoil

\[ \Rightarrow \text{Class of sum rules for derivatives of} \ \xi \]
Familiar Sum Rules

Bjorken sum rule
\[ \rho^2 = \frac{1}{4} + 2 \sum \tau_{3/2}(1)^2 + \sum \tau_{1/2}(1)^2 + \mathcal{O}(\alpha_s) \]
- sums run over “radial excitations” (up to excitation energy cutoff)

Uraltsev sum rule
\[ \sum \tau_{3/2}(1)^2 - \sum \tau_{1/2}(1)^2 = \frac{1}{4} \]
- Valid to all orders in \( \alpha_s \)

The two combined
\[ \rho^2 = \frac{3}{4} + 3 \sum \tau_{1/2}(1)^2 + \mathcal{O}(\alpha_s) \]
New Sum Rules (Le Yaouanc et al.)

At tree level:

\[ \rho^2 = \frac{12}{5} \left[ \sum \tau_{1/2}(1)\tau'_{1/2}(1) - \sum \tau_{3/2}(1)\tau'_{3/2}(1) \right] \]

\[ \sigma^2 = \frac{5}{4} \rho^2 - 3 \sum \tau_{1/2}(1)\tau'_{1/2}(1) > \frac{15}{16} \]

\[ \sigma^2 = \frac{4}{5} \rho^2 + \frac{3}{5} \sum \xi'(1)^2 > \frac{4}{5} \rho^2 + \frac{3}{5} (\rho^2)^2 > \frac{15}{16} \]

Interesting cross-checks with $D^{**}$ data?

Measure $\tau_{1/2}$ with narrow $D_s^{**}$ states (SU(3))?
## Comparisons

<table>
<thead>
<tr>
<th></th>
<th>$B \rightarrow D\ell\bar{\nu}$</th>
<th>$B \rightarrow D^*\ell\bar{\nu}$</th>
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</thead>
<tbody>
<tr>
<td><strong>Sum Rules</strong></td>
<td>$0.6 &lt; \rho_D^2 &lt; 1.0$</td>
<td>$0.5 &lt; \rho_{A_1}^2 &lt; 1.0$</td>
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<tr>
<td></td>
<td>$\sigma_D^2 &gt; 0.95$</td>
<td>$\sigma_{A_1}^2 &gt; 0.7$</td>
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<tr>
<td><strong>Disper. Rels.</strong></td>
<td>$-0.17 &lt; \rho_D^2 &lt; 1.51$</td>
<td>$-0.14 &lt; \rho_{A_1}^2 &lt; 1.54$</td>
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<tr>
<td></td>
<td>$\sigma_D^2 = 2.09\rho_D^2 - 0.31$</td>
<td>$\sigma_{A_1}^2 = 2.16\rho_{A_1}^2 - 0.47$</td>
</tr>
<tr>
<td><strong>Belle $\rho_X^2$</strong></td>
<td>$1.12 \pm 0.22 \pm 0.14$</td>
<td>$1.35 \pm 0.17 \pm 0.19$</td>
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<tr>
<td><strong>Belle $\sigma_X^2$</strong></td>
<td>$2.06 \pm 0.46 \pm 0.29$</td>
<td>$2.44 \pm 0.37 \pm 0.41$</td>
</tr>
<tr>
<td><strong>Babar $\rho_X^2$</strong></td>
<td>?</td>
<td>$1.23 \pm 0.02 \pm 0.28$</td>
</tr>
<tr>
<td><strong>Babar $\sigma_X^2$</strong></td>
<td>?</td>
<td>$2.18 \pm 0.04 \pm 0.60$</td>
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</table>
But...

Sum rule bounds for $\sigma_X^2$ were derived from Bjorken bound

Using average measured value of $\rho_X^2$ gives

- $\sigma_D^2 > 2.0$
- $\sigma_{A_1}^2 > 2.2$
Conclusions

- Sum rule constraints potentially competitive
- With $B \rightarrow D^\ast$ input, sum rules could yield much stronger constraints