

Calculations of $\text{BR}[\bar{B} \rightarrow X_s \gamma]$

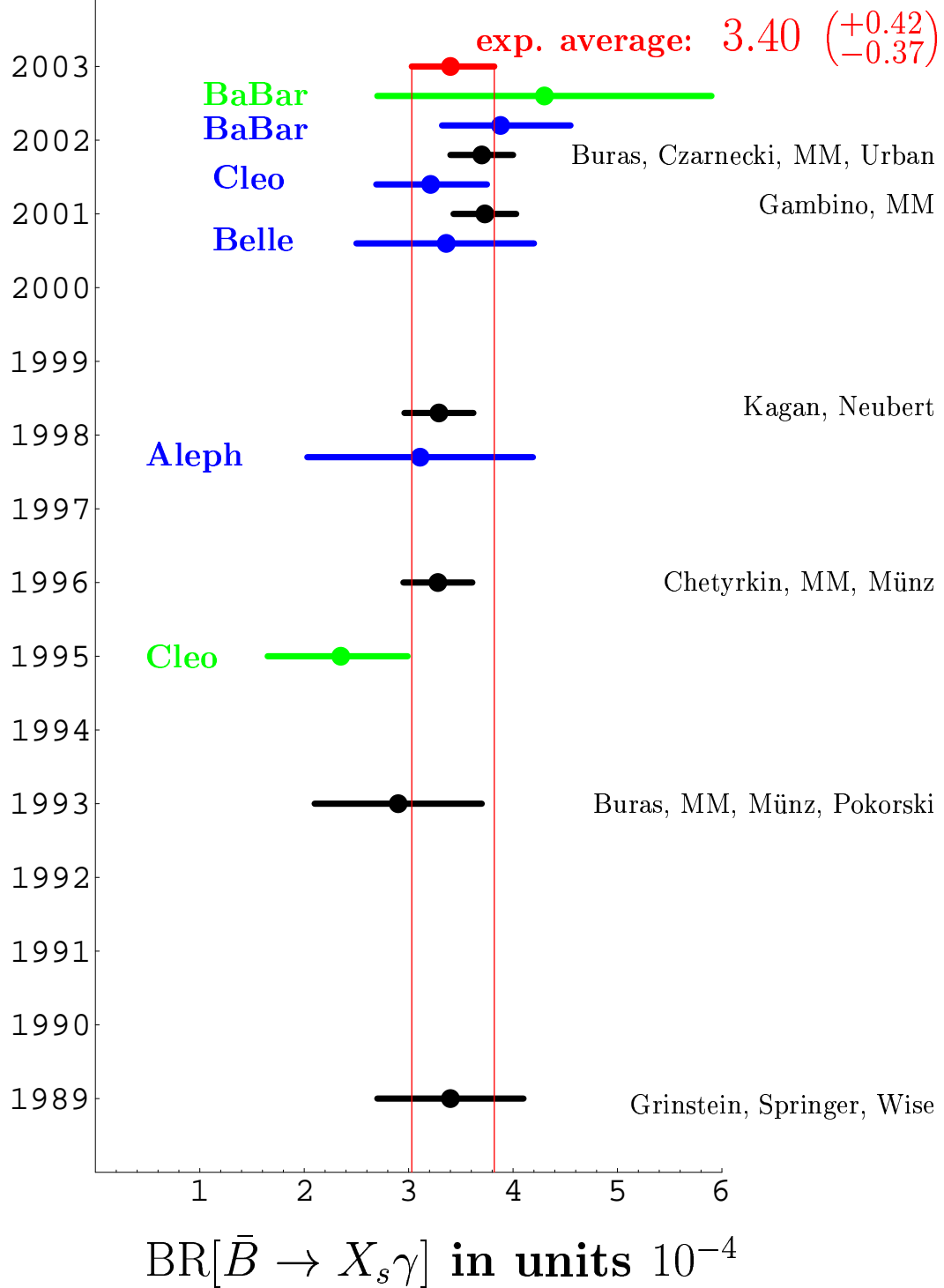
Mikołaj Misiak, Warsaw University

1. SM predictions for $\text{BR}[\bar{B} \rightarrow X_s \gamma]$ vs. experiment
2. Perturbative calculations of $b \rightarrow X_s^{\text{parton}} \gamma$
 - (i) completed (**NLO**)
 - (ii) future \leftrightarrow charm quark loops (**NNLO**)
3. Non-perturbative effects

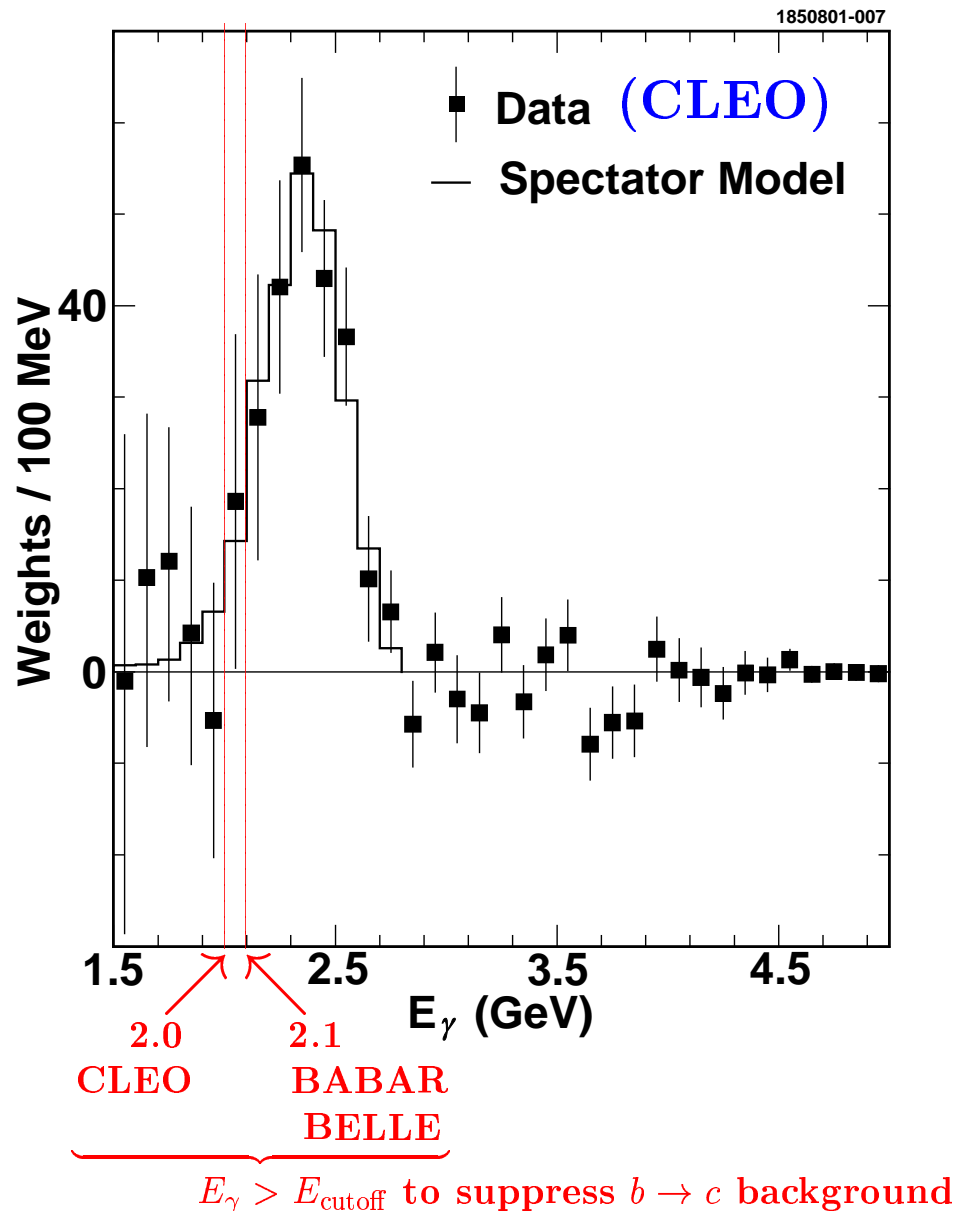
Starting point:

$$\begin{aligned}\Gamma[\bar{B} \rightarrow X_s \gamma] &\simeq \Gamma[b \rightarrow X_s^{\text{parton}} \gamma] \\ &\equiv \Gamma[b \rightarrow s \gamma] + \Gamma[b \rightarrow s \gamma g] + \dots\end{aligned}$$

Measurements and the SM calculations (some of them)



$\bar{B} \rightarrow X_s \gamma$ photon spectrum

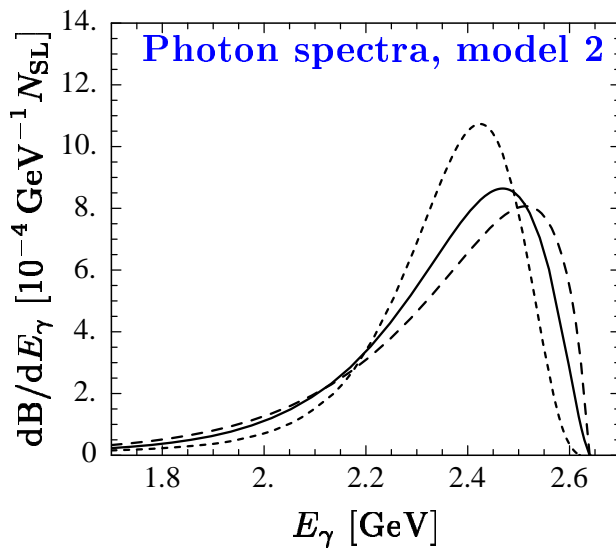
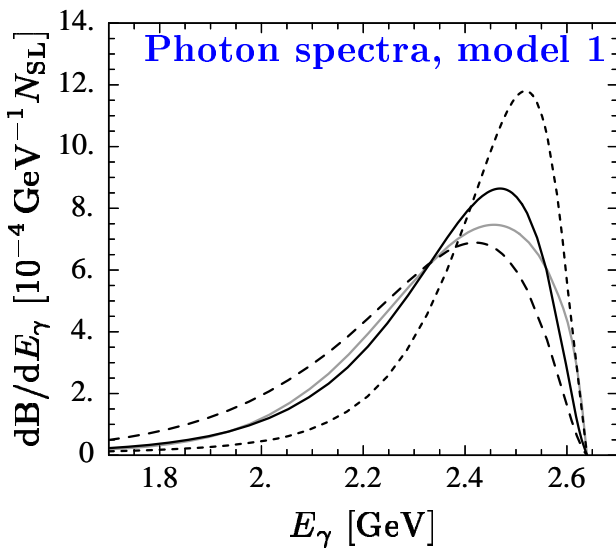
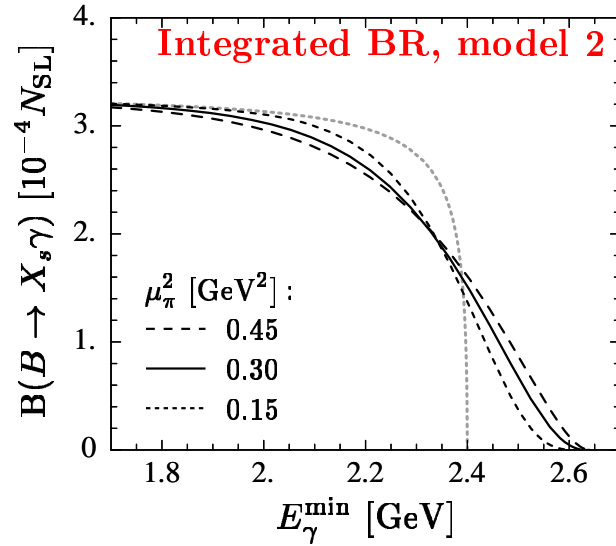
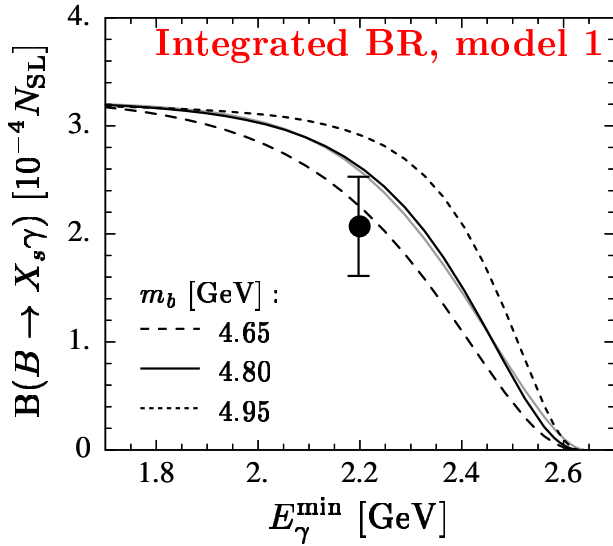


Extrapolation from the experimental cutoff to lower energies \Rightarrow model-dependence.

CLEO: $\sim \pm 5\%$, BABAR: $\sim \pm 9\%$, BELLE: $\sim \pm 15\%$

The measured spectrum is used for extraction of $\bar{\Lambda}$ and λ_1 parameters of HQET \Rightarrow determination of V_{ub} .

Model predictions for the **integrated $\bar{B} \rightarrow X_s \gamma$ branching ratio** and for the corresponding **photon spectra**.



From: **A. Kagan and M. Neubert, hep-ph/9805303, Eur. Phys. J. C7 (1999) 5.**

Problems for low E_γ :

(i) Real $c\bar{c}$ bound states inside X_s ?

(ii) Photons from light-quark fragmentation ?

[A. Kapustin, Z. Ligeti and H.D. Politzer, PLB 357 (1995) 653]

Experimental average:

$$\text{BR}[\bar{B} \rightarrow X_s \gamma \quad (E_\gamma > \frac{1}{20}m_b)]_{\text{exp}} = [3.40 \quad \begin{matrix} +0.42 \\ -0.37 \end{matrix}] \times 10^{-4}$$

After rescaling back to 1.6 GeV, one obtains

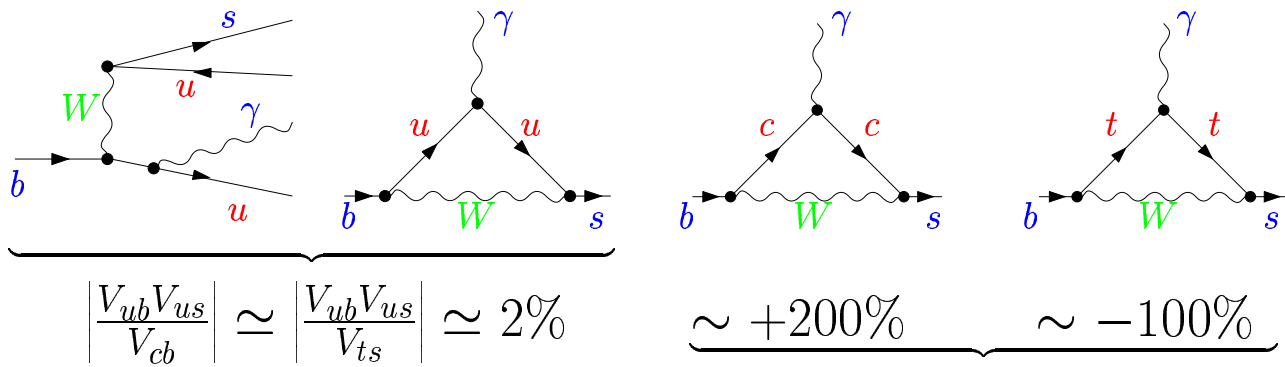
$$\text{BR}[\bar{B} \rightarrow X_s \gamma \quad (E_\gamma > 1.6 \text{ GeV})]_{\text{exp}} = [3.28 \quad \begin{matrix} +0.41 \\ -0.36 \end{matrix}] \times 10^{-4}$$

SM prediction:

$$\text{BR}[\bar{B} \rightarrow X_s \gamma]_{E_\gamma > E_0}^{\text{subtr. } \psi} = \begin{cases} (3.57 \pm 0.30) \times 10^{-4}, & E_0 = 1.6 \text{ GeV} \\ 3.70 \times 10^{-4}, & E_0 = \frac{1}{20}m_b \end{cases}$$

The dominant uncertainty ($\sim \pm 6\%$) originates from m_c -dependence.

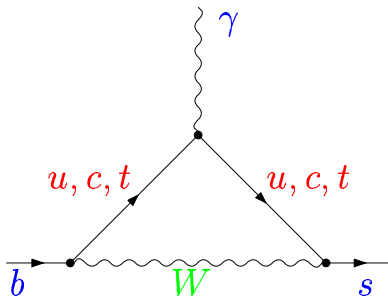
Electroweak transitions mediating $\bar{B} \rightarrow X_s \gamma$:



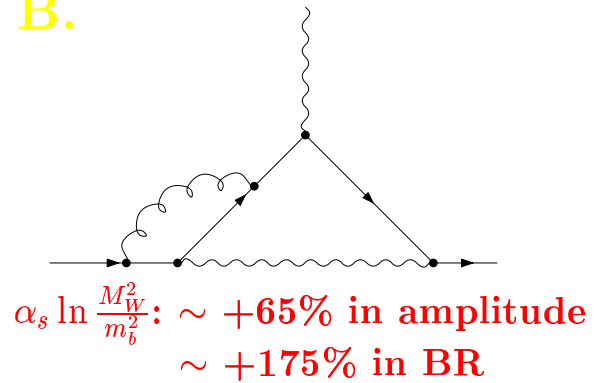
In the amplitude, after including LO QCD effects.

Examples of Feynman diagrams contributing to $b \rightarrow s \gamma$ at various orders in the renormalization-group-improved perturbation theory:

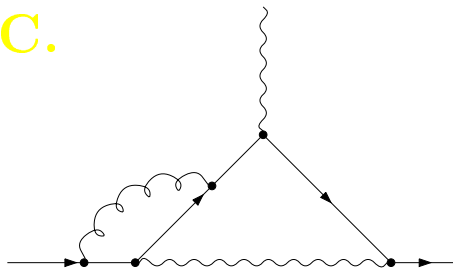
A.



B.

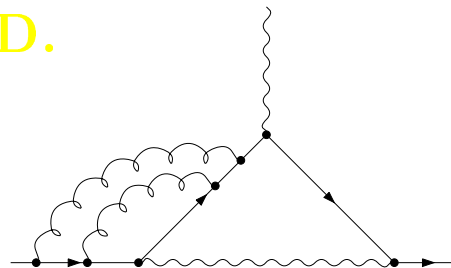


C.



non - logarithmic

D.



logarithmic

$\sim +20\%$ in BR

The effective Lagrangian:

$$\mathcal{L} = \mathcal{L}_{QCD \times QED}(u, d, s, c, b) + \frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^8 C_i(\mu) O_i$$

$$O_i = \begin{cases} (\bar{s}\Gamma_i c)(\bar{c}\Gamma'_i b), & i = 1, 2, & |C_i(m_b)| \sim 1 \\ (\bar{s}\Gamma_i b)\Sigma_q(\bar{q}\Gamma'_i q), & i = 3, 4, 5, 6, & |C_i(m_b)| < 0.07 \\ \frac{em_b}{16\pi^2} \bar{s}_L \sigma^{\mu\nu} b_R F_{\mu\nu}, & i = 7, & C_7(m_b) \sim -0.3 \\ \frac{gm_b}{16\pi^2} \bar{s}_L \sigma^{\mu\nu} T^a b_R G_{\mu\nu}^a, & i = 8, & C_8(m_b) \sim -0.15 \end{cases}$$

Three steps of the calculation:

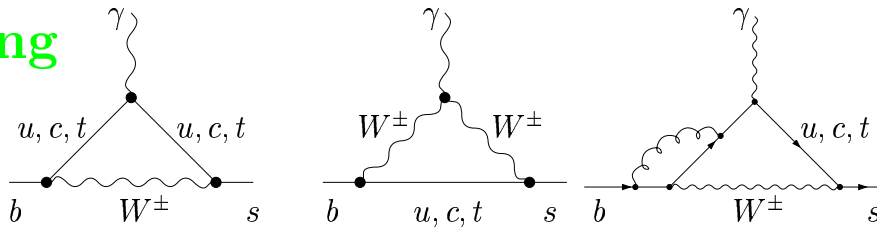
Matching: Evaluating $C_i(\mu_0)$ at $\mu_0 \sim M_W$ by requiring equality of the SM and eff. theory Green functions.

Mixing: Deriving the eff. theory RGE and evolving $C_i(\mu)$ from μ_0 to $\mu_b \sim m_b$.

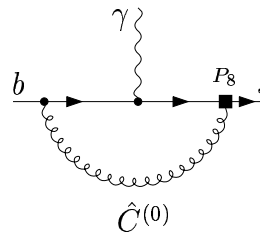
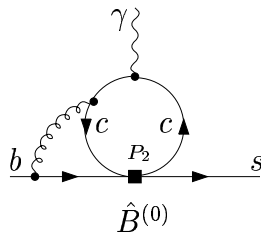
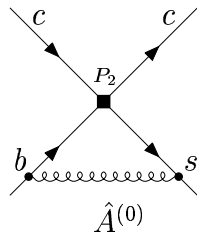
Matrix elements: Evaluating the on-shell amplitudes at $\mu_b \sim m_b$.

Examples of diagrams that have been calculated at LO and NLO:

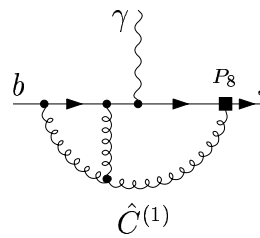
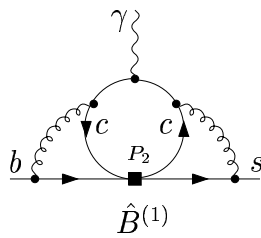
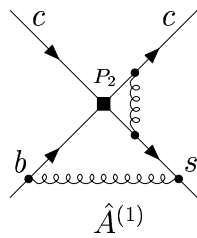
Matching



Mixing

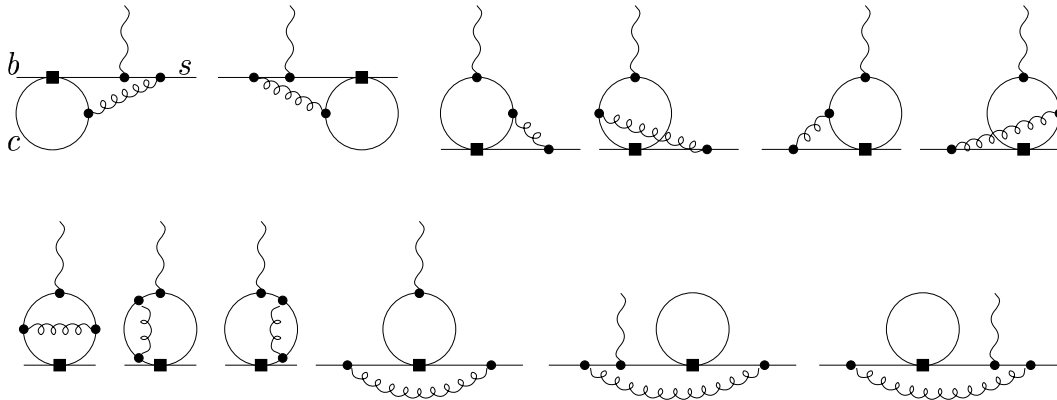


(LO)



(NLO)

Matrix elements:



At NNLO – one gluon more in each case.

Three methods have been used for calculating 2-loop matrix elements with charm-quark loops:

1. Mellin-Barnes transform of Feynman-parameter integrals \Rightarrow Expansion in m_c/m_b

[Greub, Hurth, Wyler, 1996]

2. Asymptotic expansions \Rightarrow Expansion in m_c/m_b

[Buras, Czarnecki, Misiak, Urban, 2001]

3. “Brute force” \Rightarrow **No** expansion in m_c/m_b

[Buras, Czarnecki, Misiak, Urban, 2002]

Goal: Diagrams with b -quark loops
 \Rightarrow Formally complete NLO calculation

The $b \rightarrow s\gamma$ amplitude becomes dependent on m_c only at the NLO, via 2-loop diagrams.

The question whether we should use:

$$\frac{m_c^{\text{pole}}}{m_b^{\text{pole}}} = 0.29 \pm 0.02 \quad \text{or} \quad \frac{m_c^{\overline{\text{MS}}}(\mu)}{m_b^{\text{pole}}} = 0.22 \pm 0.04$$

$\mu \in [m_c, m_b]$

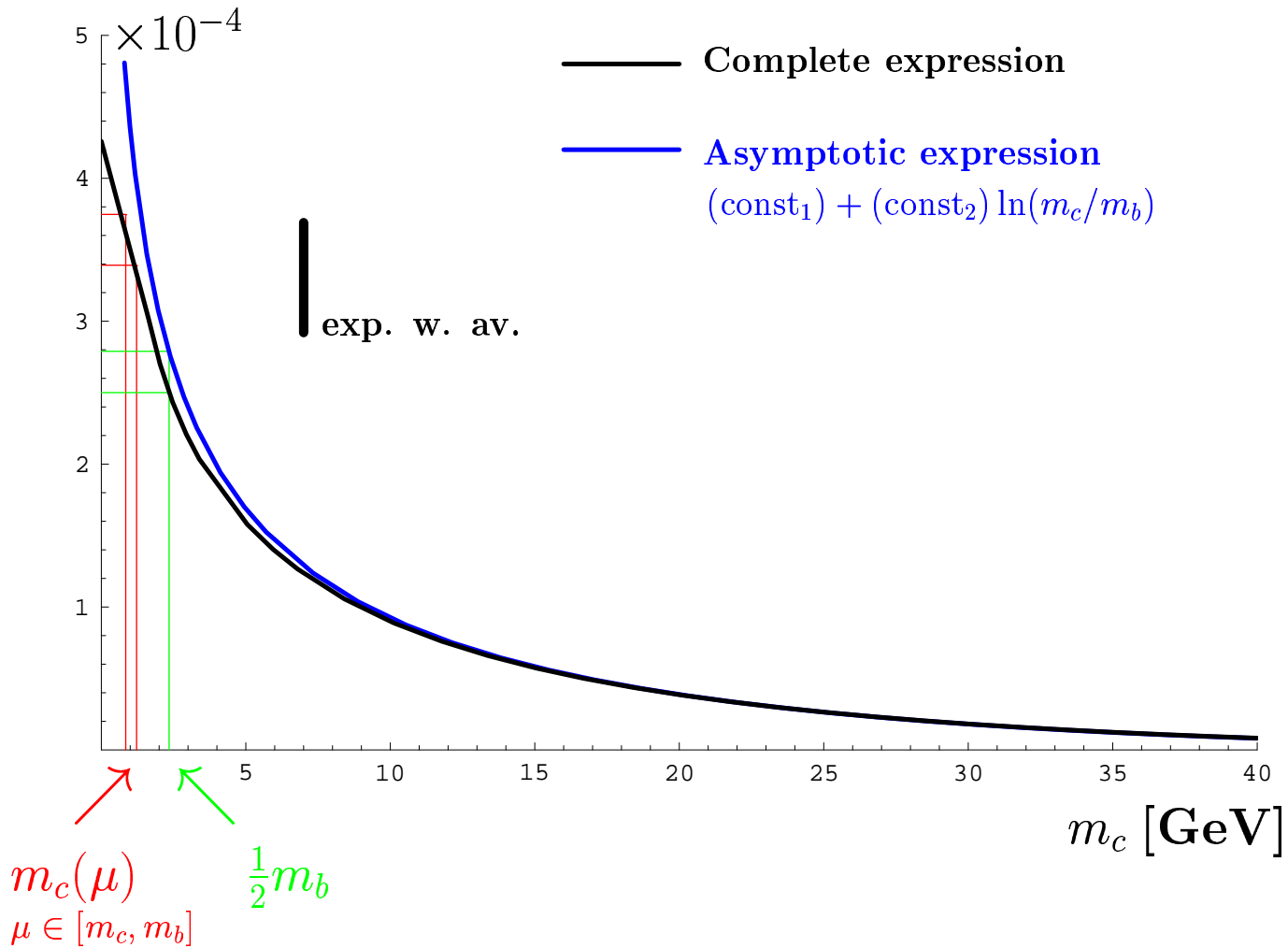
can, in principle, be asked only at NNLO.

However, changing m_c/m_b from 0.29 to 0.22 enhances $\text{BR}[\bar{B} \rightarrow X_s \gamma]$ by 10% ! Accuracy \searrow

Way out: NNLO calculation

Charm mass dependence of

$$\text{BR}[\bar{B} \rightarrow X_s \gamma \quad (E_\gamma > 1.6 \text{ GeV})]$$



⇒ Hint for the NNLO: large m_c expansion + extrapolation.

Non-perturbative effects in $\bar{B} \rightarrow X_s \gamma$

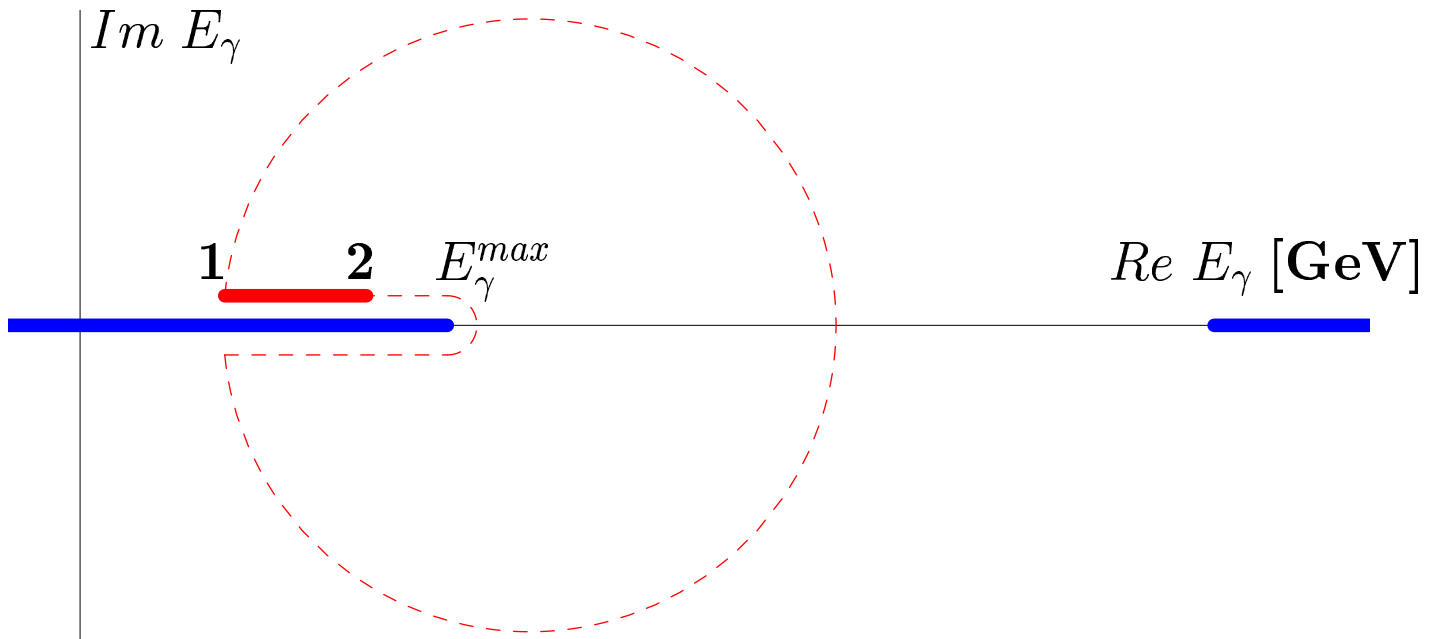
We need to sum the matrix elements of the effective Hamiltonian:

$$\Sigma_{X_s} |C_7 \langle X_s \gamma | O_7 | \bar{B} \rangle + C_2 \langle X_s \gamma | O_2 | \bar{B} \rangle + \dots|^2$$

The “77” term in the above sum can be related via optical theorem to the imaginary part of the elastic forward scattering amplitude:

$$\text{Im} \left\{ \begin{array}{c} \gamma \\ \swarrow q \quad \searrow q \\ \bar{B} \quad \bullet \quad \bullet \quad \bar{B} \\ \gamma \end{array} \right\} \equiv \text{Im} A$$

In this amplitude, we can perform **OPE** when the photons are soft enough, i.e. when $|m_B - 2E_\gamma| \gg \Lambda_{QCD}$.



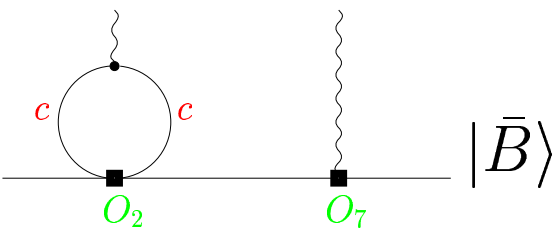
$$\int_{1 \text{ GeV}}^{E_\gamma^{\max}} dE_\gamma E_\gamma^n \text{Im} A(E_\gamma) \sim \oint_{\text{big circle}} dE_\gamma E_\gamma^n A(E_\gamma)$$

HQET gives us a double expansion:

$$\Sigma_{X_s} \text{BR}[\bar{B} \rightarrow X_s \gamma]_{E_\gamma > 1 \text{ GeV}} = \left[a_{00} + a_{02} \left(\frac{\Lambda}{m_B} \right)^2 + \dots \right] + \frac{\alpha_s(m_b)}{\pi} \left[a_{10} + a_{12} \left(\frac{\Lambda}{m_B} \right)^2 + \dots \right] + \mathcal{O} \left[\left(\frac{\alpha_s(m_b)}{\pi} \right)^2 \right]$$

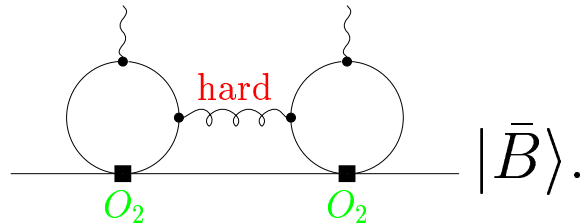
+ [Contributions other than the “77” term].

There is no OPE for the latter term. However, operators containing no charm quark are suppressed by their small Wilson coefficients. As far as the operators containing the charm quark are concerned, we know that their contribution at the leading order in α_s can be expressed as a power series:

$$\langle \bar{B} | \text{---} \overset{c}{\circlearrowleft} \overset{c}{\circlearrowright} \text{---} \overset{\gamma}{\text{---}} | \bar{B} \rangle = \frac{\Lambda^2}{m_c^2} \sum_{n=0}^{\infty} b_n \left(\frac{m_b \Lambda}{m_c^2} \right)^n,$$


which can be truncated to the leading $n = 0$ term, because the coefficients b_n decrease fast with n . The calculable $n = 0$ term makes $\text{BR}[\bar{B} \rightarrow X_s \gamma]$ increase by around 3%.

At $\mathcal{O}(\alpha_s)$, one encounters matrix elements like:

$$\langle \bar{B} | \text{---} \overset{\gamma}{\circlearrowleft} \text{---} \overset{\text{hard}}{\text{---}} \text{---} \overset{\gamma}{\circlearrowright} \text{---} | \bar{B} \rangle.$$


Perturbative contributions to such matrix elements are usually small, so it is enough to rely on quark-hadron duality and use the perturbative NLO expressions only. However, the intermediate ψ and ψ' contributions ($\bar{B} \rightarrow \psi^{(\prime)} X$ followed by radiative $\psi^{(\prime)}$ decay) must be subtracted on the experimental side.

Summary:

1.

$$BR[\bar{B} \rightarrow X_s \gamma]_{E_\gamma > E_0} = \begin{cases} (3.57 \pm 0.30) \times 10^{-4}, & E_0 = 1.6 \text{ GeV} \\ 3.70 \times 10^{-4}, & E_0 = \frac{1}{20} m_b \end{cases}$$

$$BR[\bar{B} \rightarrow X_s \gamma]_{\text{exp}}^{E_0 = \frac{1}{20} m_b} = \left[3.40 \begin{pmatrix} +0.42 \\ -0.37 \end{pmatrix} \right] \times 10^{-4}$$

⇒ Theory and experiment agree within 1σ

2. The above agreement provides stringent constraints on new physics.

3. The dominant theoretical uncertainty is of perturbative origin. It is due to the renormalization-scheme dependence of m_c in two-loop contributions to the decay amplitude.

Calculating the NNLO corrections would remove this problem and, in consequence, reduce the theoretical uncertainty by almost a factor of 2. The main challenge at the NNLO are UV-finite parts of 3-loop massive vertex integrals with non-vanishing external momenta.

4. The numerical behaviour of $BR[\bar{B} \rightarrow X_s \gamma]$ as a function of m_c suggests that the dominant charm mass dependence originates from distances much smaller than Λ_{QCD} . In such a case, the associated non-perturbative effects would be under control.