A very Short Introduction to the Soft-Collinear Effective Theory

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$|V_{xb}|$ and $|V_{tx}|$
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SCET

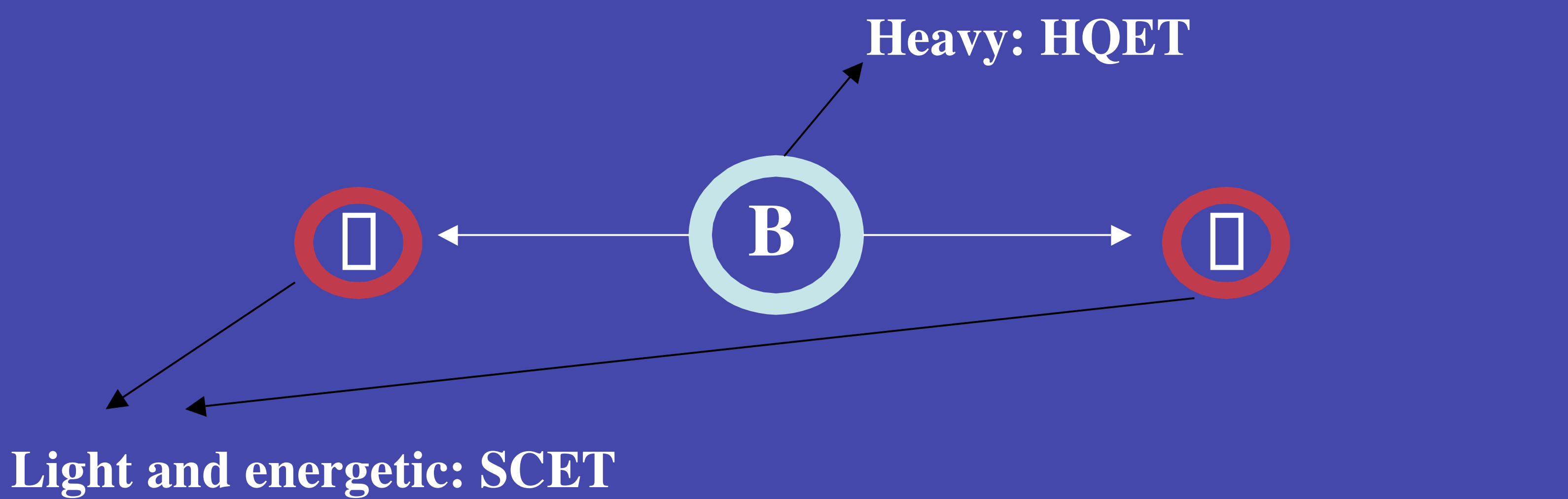
Bauer, Fleming, Luke, Phys. Rev. D 63: 014006, 2001
Bauer, Fleming, Pirjol, Stewart, Phys. Rev. D 63: 114020, 2001
Bauer & Stewart, Phys. Lett. B 516: 134, 2001
Bauer, Pirjol, Stewart, Phys. Rev. D 65: 054022, 2002

Effective field Theory of highly energetic particles that have a small invariant mass

- **E >> M: Near the lightcone**
  - $p = (p^+, p^-, p_\perp) \sim Q(M^2/Q^2, 1, M/Q) \sim Q(\lambda^2, 1, \lambda)$
  - $\lambda \ll 1$, and $p^2 \sim \lambda^2$
- **SCET has the right degrees of freedom for describing energetic particles interacting with soft "stuff"**

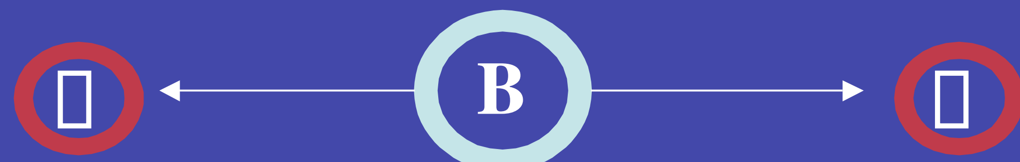
Analogous to HQET: Effective Field Theory of Heavy and soft degrees of freedom--describes heavy particles interacting with soft "stuff"

If you only remember one thing… Remember this picture:



SCET describes the light and energetic particles
SCET is QCD in a limit

Kinematics



Pion momentum: $p_\pi^\mu = (2.640\ \text{GeV},0,0,-2.636\ \text{GeV})$

$\approx Q\, n^\mu \qquad n^\mu = (1,0,0,-1)$

$= (0,2,0_\perp)$ ⇐ LC coordinates

- **Corrections are small ~ Λ_{QCD} , m_π relative to Q**
  - **Expansion in** $\frac{\Lambda_{QCD}}{Q}$ **or** $\sqrt{\frac{\Lambda_{QCD}}{Q}}$

Motivation

- **Systematic: power counting in small parameter □**
- **Understand Factorization in a universal way**
  - **Key to separate hard contributions from soft & collinear**
  - **Systematic corrections to factorization (power counting)**
- **Symmetries**
  - **Reduce the number form-factors**
  - **In HQET where there is only the Isgur-Wise function**
- **Sum infrared logarithms**
  - **Sudakov logarithms**

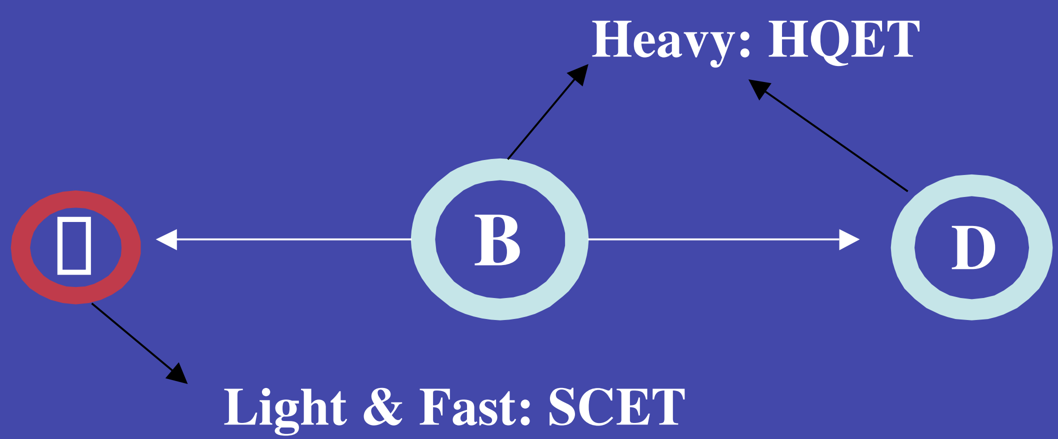
So…what’s it good for?

SCET couple to HQET can be used for any decays involving stationary heavy, and fast light particles:

B □ D□,
B □ □e□, B □ □e□, B □ K^* □, B □ K e^+ e^- ,
B □ □□, B □ K□,
B □ X_u e □, B □ X_s□, □ □ X □…
DIS, Drell Yan,
□*□□ $□^0$,□□ □ □, …

$B \longrightarrow D\pi$ factorization

J.D. Bjorken: Color-transparency, Nucl. Phys. B (Proc. Suppl.) 11, 1989, 325
Dugan & Grinstein: Factorization in LEET Phys. Lett. B255: 583, 1991
Politzer & Wise: Factorization (proposed) Phys. Lett. B257: 399, 1991
Beneke, Buchalla, Neubert, Sachrajda: QCD factorizaton (proved to 2 loops) Nucl. Phys. B591: 313, 2000
Bauer, Pirjol, Stewart: SCET (proved to all orders in α_s) Phys. Rev. Lett. 87: 201806, 2001



$$\frac{i}{2}\, m_b\, E_\pi\, f_\pi\, F^{B\to D}(0) \int dx\; T(x,\mu)\; \phi_\pi(x,\mu)$$

Soft B→ D form factor

Hard coefficient calculate in PT: $\alpha_s(M_b)$

Light-cone pion wavefunction: nonperturbative

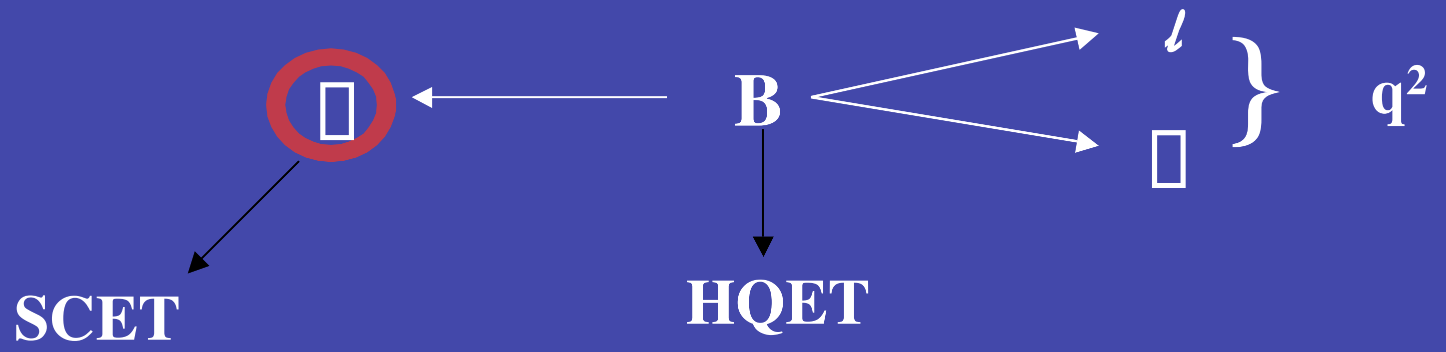
Semi-leptonic heavy-to-light

Selected history:

Brodsky *et. al*. (1990)	Hard part, $1/x^2$ singularity
Li & Yu (1996)	k_T factorization, Sudakov suppression
Bagan, Ball, Braun (1997)	Light-cone sum rules
Charles *et. al*. (1998)	Symmetry relations: □(E), $□_{□}$(□), $□_{\|\|}$(□)
Beneke & Feldman (2000)	$\mathcal{O}(□_s)$ corrections, factorization proposal
Bauer *et. al*. (2000)	Collinear gluons, $C_i(\mathcal{P})$, soft factorization
Descotes, Sachradja (2001)	More on Sudakov suppression
Bauer, Pirjol, Stewart (2002)	Factorization in SCET
Pirjol & Stewart (2002)	Details of factorization in SCET

Semi-leptonic heavy-to-light

e.g. B □ □ l □ at large recoil



SCET factorization: all orders in $\square_s$, leading order in □: Bauer, Pirjol. Stewart: hep-ph/0211069

$$F(M^2) = \frac{1}{2}\, f_B\, f_M \int dz \int dx \int dr_+\, T(z,M,\square_0)\, J(z,x,r_+,M,\square_0,\square)\, \square_{\square}(x,\square)\, \square_{\square}^{+}(r_+,\square)$$

$$+\, C_k(M,\square)\, \square_k(q,\square)$$

Factorizable piece
Non-perturbative parameters:
decay constants, LC wave functions

Non-factorizable piece
Non-perturbative form factors
(restricted by symmetries in SCET)

- **Note both the pieces are same order in power counting!**

B □ □ l □: Q^2 range where SCET is valid

- $m_{□}$ = 770 Mev
- Remember for SCET to be valid we need Q >> $□_{QCD}$, $m_{□}$

Q^2 (GeV^2)	E (GeV)	P (GeV)	$m_{□}$/□	or $m_{□}$/2 □
0	2.70	2.58	0.286	0.143
0.25	2.67	2.56	0.288	0.144
1	2.60	2.48	0.300	0.150
2.25	2.48	2.36	0.310	0.155
4	2.32	2.19	0.330	0.165
6.25	2.10	1.96	0.360	0.180

Too Big?!?!

Heavy-to-light factorization in SCET: Details

$$F(Q^2) = \frac{1}{2} f_B f_M \int dz \int dx \int dr_+ \, T(z,Q,\square_0)\, J(z,x,r_+,Q\,\square_0,\square)\, \square_\square(x,\square)\, \square_\square^{+}(r_+,\square)$$

Decay constants ($f_B f_M$)

Calculable ($T(z,Q,\square_0)\, J(z,x,r_+,Q\,\square_0,\square)$)

Light-cone wave-functions ($\square_\square(x,\square)\, \square_\square^{+}(r_+,\square)$)

$$+\, C_k(Q,\square)\, \square_k(Q,\square)$$

Calculable ($C_k(Q,\square)$)

Soft form factor ($\square_k(Q,\square)$)

- $T(z,Q,\square_0)$ & $C_k(Q,\square)$: **Expansion in** $\square_s(Q)$
- $J(z,x,r_+,Q\,\square_0,\square)$: **Expansion in** $\square_s(\overline{Q\square})$

} $Q \sim \{m_b, E = m_b - q^2/(2m_b)\}$

Factorization in B □ □□(□)

QCD Factorization Proposed: Beneke, Buchalla, Neubert, Sachrajda: Phys. Rev. Lett. 83: 1914, 1999
Nucl. Phys. B591: 313, 2000

$$F(M) = f^{B\Box}\,\Box(0) \int dx\, T^{I}(x)\,\Box_{\Box}(x) + \int d\Box\, dx\, dy\, T^{II}(\Box, x, y)\,\Box_{\Box}(x)\,\Box_{\Box}(y)$$

- Was shown to hold to order $\Box_s$

Perturbative QCD: Keum, Li, Sanda: hep-ph/0201103

$$F(M) = 0 + \int d\Box\, dx\, dy\, T^{II}(\Box, x, y)\,\Box_{\Box}(x)\,\Box_{\Box}(y)$$

- **Sum Sudakov logarithms**

No proof in SCET yet

- **It is not a given that this will give the above formula**
- **Wait and see…**

What's to come ?

- **Proof of factorization in B □ □□**
  - **Phenomenology**

- **Phenomenology in heavy-to-light semileptonic decays**
  - **Forward backward asymmetry**
  - **Extraction of form factors**
  - **Decay rates**