V_{ub} without shape functions (really!)

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- Introduction

  ... $|V_{ub}|$ is very important to overconstrain CKM

  "It doesn't really matter in this program whether you measure a CP violating quantity or not. The length of a side is as good as an angle." (M. Wise, hep-ph/0111167)

- Semileptonic $B \to X_u \ell \bar{\nu}$ rate and spectra

  ... Optimal cuts to eliminate $b \to c$ background

- Few comments on nonleptonic $B \to X_{\not{c}\not{s}}$ rate (⇒ Campagnari)

- Summary

- I will not talk about: Exclusive decays (⇒ Postler / Kronfeld)

  Lepton endpoint (⇒ Rothstein)

Inclusive semileptonic B decay

- Operator Product Expansion (OPE): expand decay rates in $\Lambda_{\rm QCD}/m_b$ and $\alpha_s(m_b)$

  $\Rightarrow$ model independent results for "sufficiently inclusive" observables

$$\mathrm{d}\Gamma = \begin{pmatrix} b\ \text{quark} \\ \text{decay} \end{pmatrix} \times \left\{ 1 + \frac{0}{m_b} + \frac{f(\lambda_1, \lambda_2)}{m_b^2} + \ldots + \alpha_s(\ldots) + \alpha_s^2(\ldots) + \ldots \right\}$$

  Interesting quantities computed to order α_s, $\alpha_s^2\beta_0$, and $1/m^2$

  ($1/m^3$ used to estimate uncertainties)

- Good news: Total rates known at few ($\lesssim 5$) percent level (duality...)

  Improvements: better m_b from Υ sum rules / moments of B decay spectra / Lattice

  Upsilon expansion: (Hoang, ZL, Manohar, PRL 82 277, PRD 59 074017)

$$|V_{ub}| = \left(3.04 \pm 0.06_{(\text{pert})} \pm 0.08_{(m_b)}\right) \times 10^{-3} \left(\frac{\mathcal{B}(B \to X_u \ell \bar\nu)}{0.001} \, \frac{1.6\,\text{ps}}{\tau_B} \right)^{1/2}$$

  Central value: 3.24 (Bigi, Shifman, Uraltsev, ARNPS 47 591); 3.08 (Uraltsev, IJMP A14 4641)

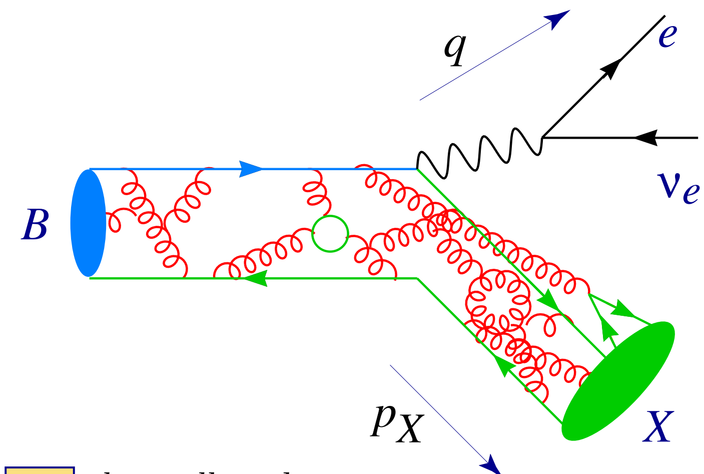
Inclusive $B \to X_u \ell \bar{\nu}$ decay and $|V_{ub}|$

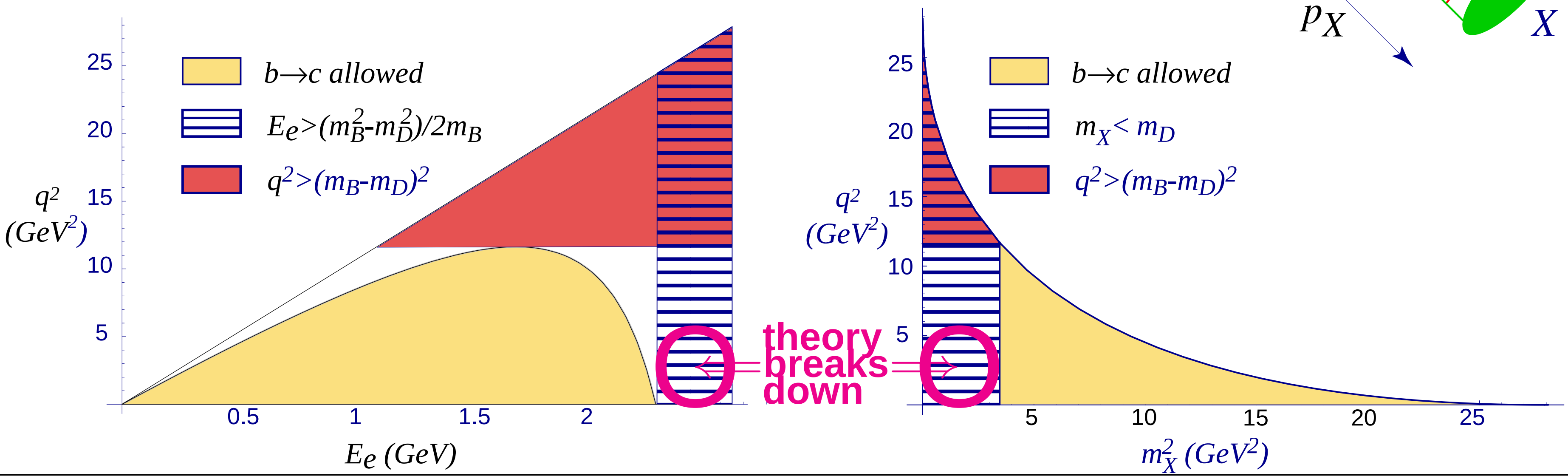
- Bad news: In certain restricted regions of phase space the OPE breaks down

Stringent cuts required to eliminate $b \to c$ background... and the troubles begin!

Proposals to measure $|V_{ub}|$:

- Lepton spectrum: $E_\ell > (m_B^2 - m_D^2)/2m_B$
- Hadronic mass spectrum: $m_X < m_D$
- Dilepton mass spectrum: $q^2 > (m_B - m_D)^2$





$B \to X_u \ell \bar\nu$ spectra

- Three qualitatively different regions of phase space:

  1) $m_X^2 \gg E_X \Lambda_{\rm QCD} \gg \Lambda_{\rm QCD}^2$: the OPE converges, first few terms can be trusted

  2) $m_X^2 \sim E_X \Lambda_{\rm QCD} \gg \Lambda_{\rm QCD}^2$: infinite set of terms equally important, the OPE becomes a twist expansion

  3) $m_X \sim \Lambda_{\rm QCD}$: resonance region — cannot compute reliably

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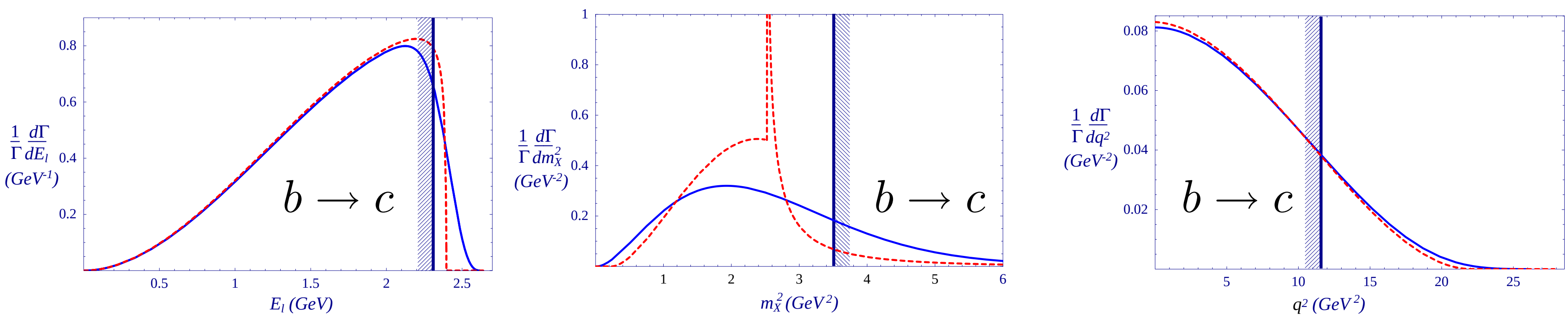
- Both $E_\ell > (m_B^2 - m_D^2)/2m_B$ and $m_X < m_D$ are in (2) since $m_B \Lambda_{\rm QCD} \sim m_D^2$



— b quark decay to $O(\alpha_s)$
— incl. “Fermi-motion” (model)

Theory happy →

← Experiment happy

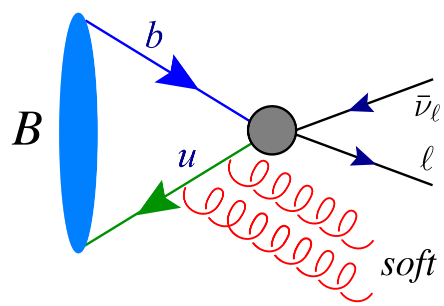
V_{ub}: q^2 spectrum

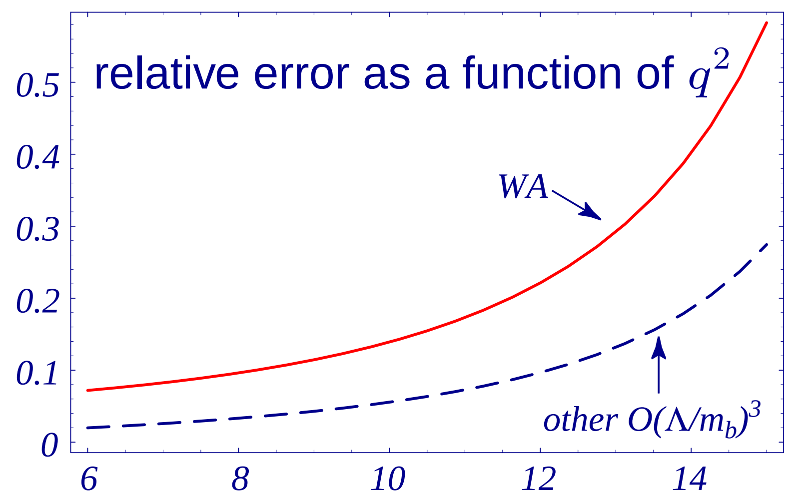
- In large q^2 region, first few terms in OPE can be trusted (Bauer, ZL, Luke, PLB 479 395)
  Reason: $q^2 > (m_B - m_D)^2$ cut implies $E_X < m_D$ [$\Rightarrow m_X^2 \gg E_X \Lambda_{\rm QCD}$]

  Leading and subleading logs of $x = m_b/(m_b - \sqrt{q^2})$ were summed ($\alpha_s^{n+1} \ln^n x$, $x\, \alpha_s^n \ln^n x$); results consistent within 1σ (Becher, Neubert, hep-ph/0105217)

Unknown corrections are $\sim O(\Lambda_{\rm QCD}/m_b)^3$
Weak annihilation dominates (Voloshin, PLB 515 74)

Guesstimate: $\sim$ 2–3% of $b \to u$ semileptonic rate; delta-function at maximal q^2 and maximal E_ℓ





$\Rightarrow$ Constrain WA by comparing D^0 vs. D_s SL widths, or V_{ub} from $B^\pm$ vs. B^0 decay

V_{ub}: combine q^2 & m_X cuts

- Can get $|V_{ub}|$ with theoretical error at 5–10% level, from up to $\sim 45\%$ of the events

(Bauer, ZL, Luke, PRD 64 113004)

Such precision can be achieved even with cuts away from the $b \to c$ threshold

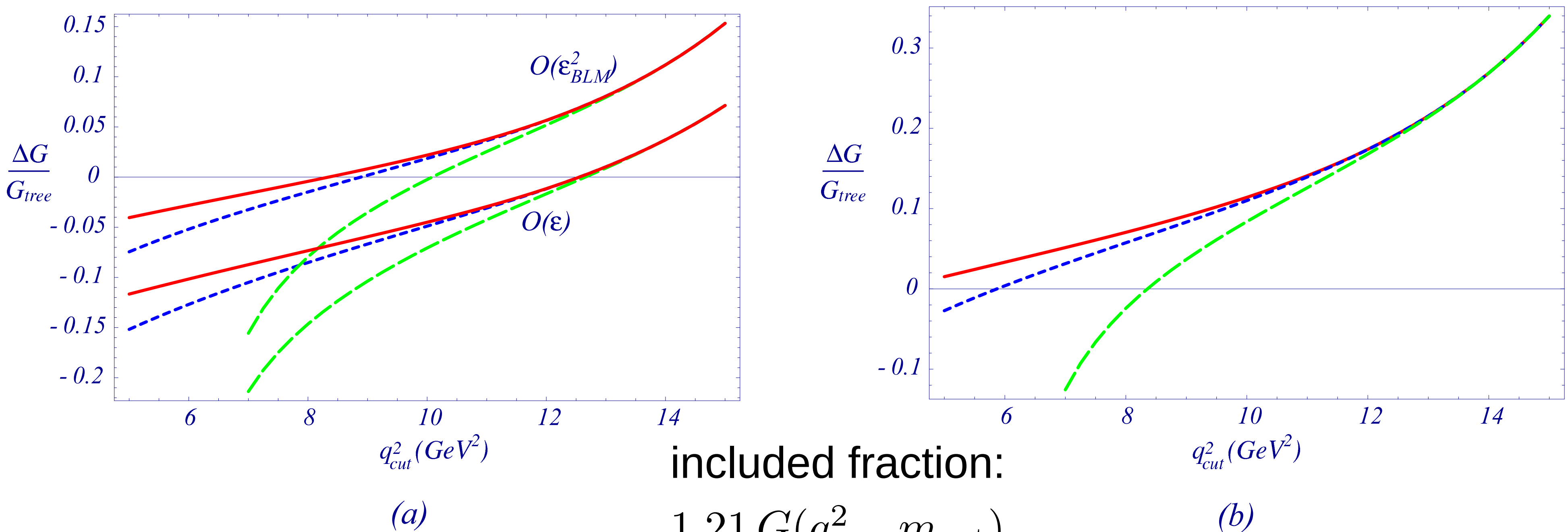
Cuts on $(q^2,\, m_X^2)$	included fraction of $b \to u\ell\bar\nu$ rate	error of $\lvert V_{ub}\rvert$ $\delta m_b = 80/30\,\mathrm{MeV}$
$6\,\mathrm{GeV}^2,\ m_D$	46%	$8\%/5\%$
$8\,\mathrm{GeV}^2,\ 1.7\,\mathrm{GeV}$	33%	$9\%/6\%$
$(m_B - m_D)^2, m_D$	17%	$15\%/12\%$

- Compared to pure q^2 cut: expansion in $\Lambda_{\rm QCD}/m_c \Rightarrow m_b\Lambda_{\rm QCD}/(m_b^2 - q_{\rm cut}^2)$

– reduced uncertainties from perturbation series and nonperturbative corrections

– uncertainty from the b quark light-cone distribution function only turns on slowly

Uncertainties (1): perturbation series



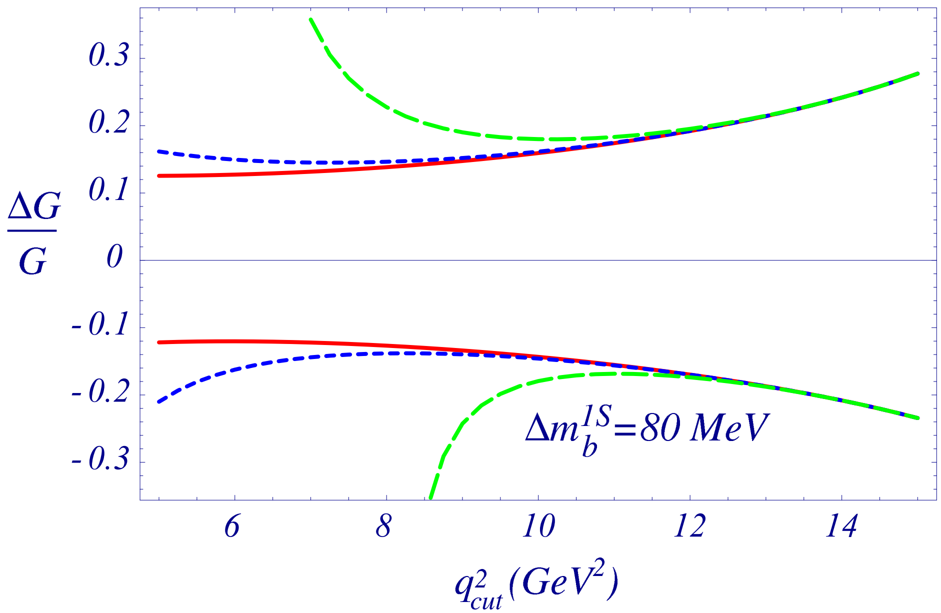
included fraction:

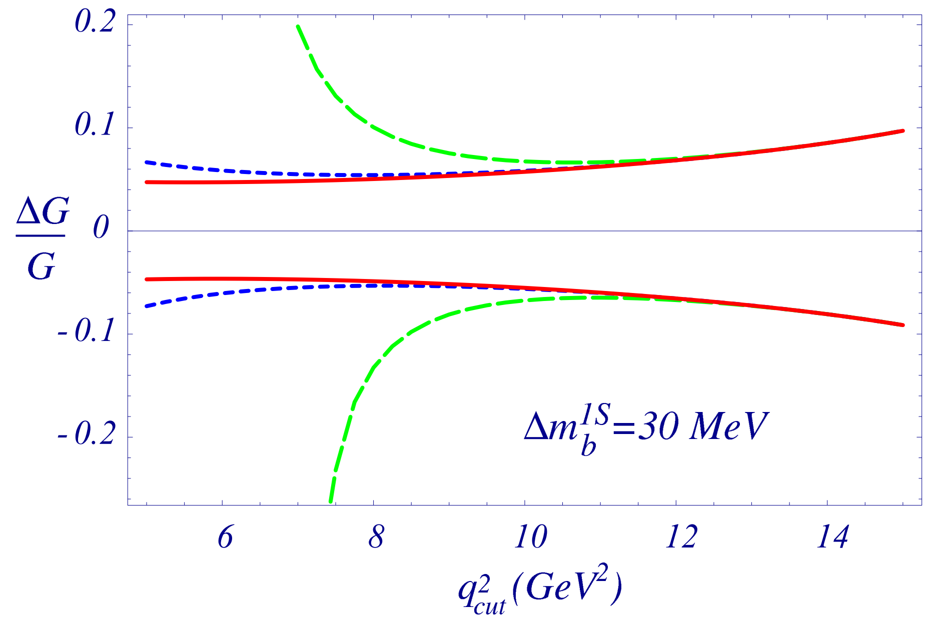
$1.21\, G(q^2_{\rm cut}, m_{\rm cut})$

(a) The $O(\epsilon)$ and $O(\epsilon^2_{\rm BLM})$ contributions to $G(q^2_{\rm cut}, m_{\rm cut})$, normalized to tree level result, for $m_{\rm cut} = 1.86\,{\rm GeV}$ (solid), $1.7\,{\rm GeV}$ (short dashed), $1.5\,{\rm GeV}$ (long dashed)

(b) Scale variation: the difference between the perturbative corrections to $G(q^2_{\rm cut}, m_{\rm cut})$, normalized to the tree level result, for $\mu = 4.7\,{\rm GeV}$ and $\mu = 1.6\,{\rm GeV}$

Uncertainties (2): b quark mass

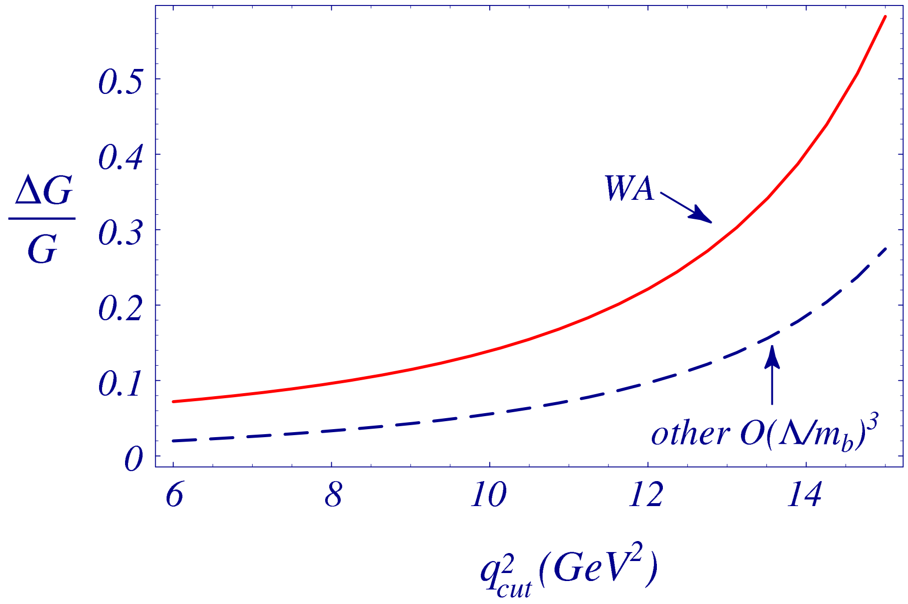




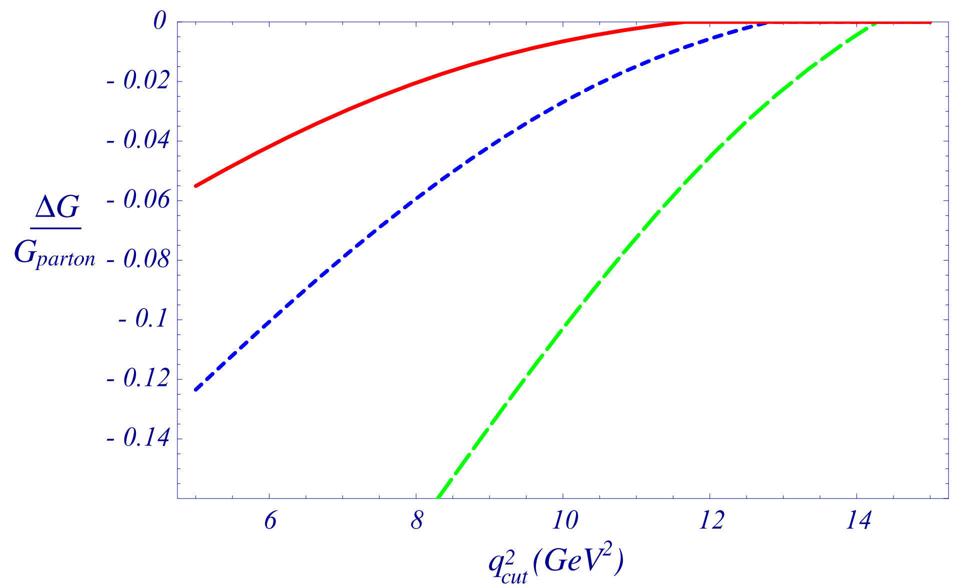
Fractional effect of $\pm 80\,\mathrm{MeV}$ (left) and $\pm 30\,\mathrm{MeV}$ (right) uncertainty in m_b^{1S} on $G(q^2_{\rm cut}, m_{\rm cut})$ for $m_{\rm cut} = 1.86\,\mathrm{GeV}$ (solid line), $1.7\,\mathrm{GeV}$ (short dashed line), and $1.5\,\mathrm{GeV}$ (long dashed line)

Uncertainties (3): higher dimension operators

(showed this before:)



Estimate of the uncertainties due to dimension-six terms in the OPE as a function of $q^2_{\rm cut}$ from weak annihilation (solid) and other operators (dashed)



Effect of a model structure function on $G(q^2_{\rm cut}, m_{\rm cut})$ as a function of $q^2_{\rm cut}$ for $m_{\rm cut} = 1.86\,{\rm GeV}$ (solid), $1.7\,{\rm GeV}$ (short dashed) and $1.5\,{\rm GeV}$ (long dashed)

$$f(k_+) = \frac{a^a}{\Gamma(a)}\,(1-x)^{a-1}\,e^{-a(1-x)}$$

a and x related to $\bar\Lambda$ and λ_1

$|V_{ub}|$ from inclusive nonleptonic decay?

Disclaimer: I had not thought much about this!

C. Campagnari, O. Long *et al.*: Can one predict $\mathcal{B}(B \to X_{\not c \not s})$?

Some issues:

- Perturbation series contains logs of m_W/m_b; probably better to work to fixed order (α_s^2) and get all terms, than to sum any series ($\exists$ in the literature)
- Measurement would eliminate $b \to u\bar u d + s\bar s$; this is tiny as predicted by the OPE + perturbation theory — makes me more and more and **more** worried!
- $b \to c\bar c d \to$ charmless: $\mathcal{B}(B \to \psi(nS)X_s) =$ few $\%$, $X_s \to X_d$ wins $\lambda^2 \sim 20$
- $b \to dg$: seems OK assuming no new physics (ask Alex Kagan)
- $b \to c\bar u d$ followed by $c \to d$: “experimental problem...”

No obvious show-stopper... [doesn’t mean that there isn’t a non-obvious one]

Summary

- Situation for $|V_{ub}|$ may become similar to present $|V_{cb}|$ — at or below 10% level:
  Inclusive: neutrino reconstruction seems crucial to obtain q^2 and m_X
  Exclusive: possibly only when unquenched lattice gets there

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- Optimized q^2 and m_X cuts can eliminate kinematically allowed region of $b \to c$ background, while keeping the theory error of $|V_{ub}|$ close to that of $\mathcal{B}(B \to X_u \ell \bar{\nu})$

Strategy: (i) reconstruct q^2 and m_X; make cut on m_X as large as possible
(ii) for a given m_X cut, reduce q^2 cut to minimize overall uncertainty

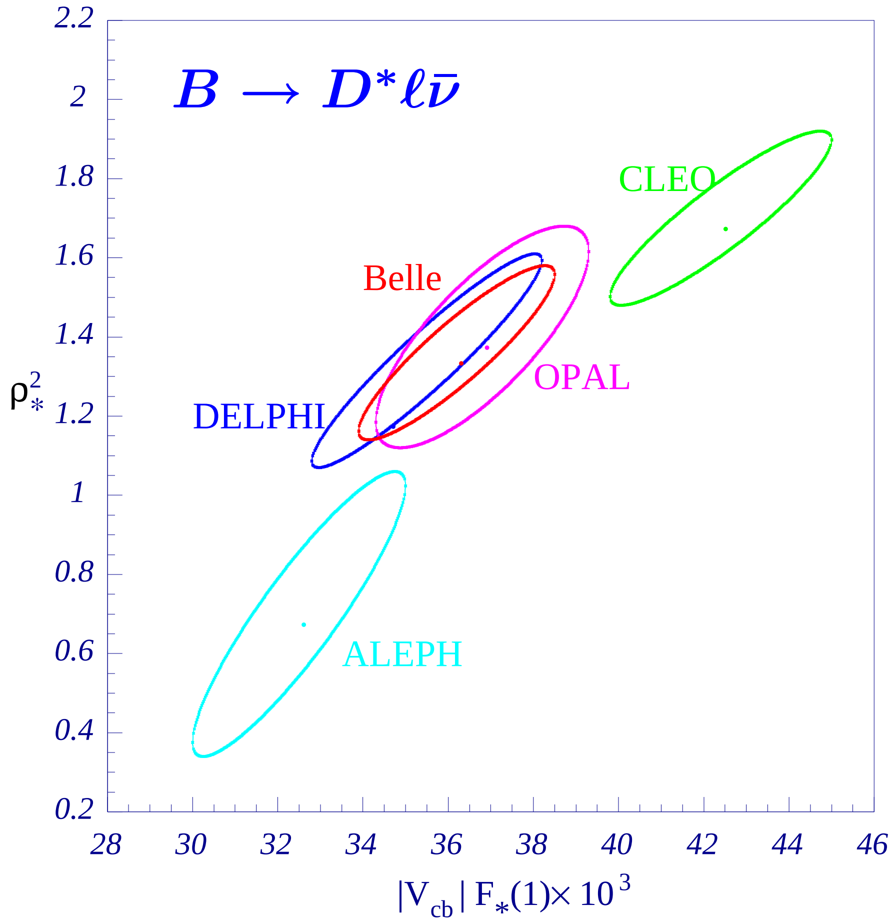
$\Rightarrow$ Would reduce SM allowed range of $\sin 2\beta$ very significantly

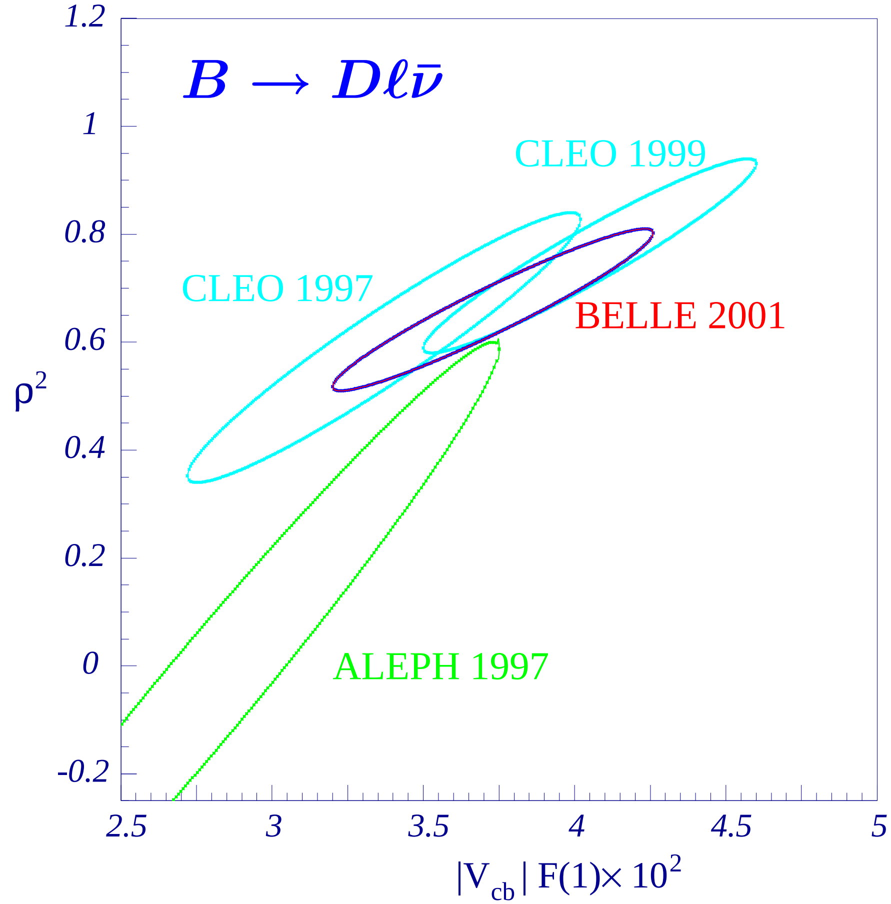
Two slides on exclusive $|V_{cb}|$

(B. Grinstein and Z.L., hep-ph/0111392)

$B \to D^{(*)}\ell\bar{\nu}$ shapes and $|V_{cb}|$ (1)

- Correlation between ρ^2 and $|V_{cb}|\,\mathcal{F}_*(1)$ is very large — assume $\mathcal{F}_{(*)}(1)$ known
  How to best use constraints from comparing shapes of $B \to D^*$ and $B \to D$?





$B \to D^{(*)}\ell\bar\nu$ shapes and $|V_{cb}|$ (2)

Predictions: [QCD SR: $\chi_2(1) \simeq -0.04,\ \chi_3'(1) \simeq 0.02,\ \eta(1) \simeq 0.6,\ \eta'(1) \simeq 0$]

$$\rho^2_{\mathcal{F}} - \rho^2_{\mathcal{F}_*} = 0.20 + 0.05\,\epsilon - 0.01\,\epsilon^2_{\rm BLM} + 0.08\,\eta(1) + 0.14\,\eta'(1)$$
$$+ 1.0\,\chi_2(1) - 3.0\,\chi_3'(1) - 0.02\,\lambda_1/{\rm GeV}^2 \simeq 0.19$$
$$\rho^2_{A_1} - \rho^2_{\mathcal{F}} = -0.12 - 0.03\,\epsilon + 0.01\,\epsilon^2_{\rm BLM} + 0.06\,\eta(1) - 0.14\,\eta'(1)$$
$$- 0.75\,\chi_2(1) + 3.0\,\chi_3'(1) + 0.01\,\lambda_1/{\rm GeV}^2 \simeq -0.02$$

Data:

Fitted slope parameter	CLEO	BELLE
$B \to D^*\ell\bar\nu$, unitarity constrained fit to $\rho^2_{A_1}$	$1.67 \pm 0.11 \pm 0.22$	$1.35 \pm 0.17 \pm 0.19$
$B \to D^*\ell\bar\nu$, linear fit to $\rho^2_{\mathcal{F}_*}$	$0.98 \pm 0.09 \pm 0.07$	$0.89 \pm 0.09 \pm 0.05$
$B \to D\ell\bar\nu$, unitarity constrained fit to $\rho^2_{\mathcal{F}}$	$1.30 \pm 0.27 \pm 0.14$	$1.16 \pm 0.25 \pm 0.15$
$B \to D\ell\bar\nu$, linear fit to $\rho^2_{\mathcal{F}}$	$0.76 \pm 0.16 \pm 0.08$	$0.69 \pm 0.14 \pm 0.09$

Ultimately want to fit: $\rho^2_{A_1},\ \rho^2_{\mathcal{F}},\ [c^2_{A_1},\ c^2_{\mathcal{F}}],\ R_1,\ R_2\ \Rightarrow\ |V_{cb}|$