# *Summary of $|V_{ub}|$ Determinations*

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$V_{xb}$ Workshop – SLAC – 7 December 2001

# *Introductory Remarks*

* $|V_{ub}|$ can be measured in semileptonic $B$ decays based on $b \to u\, l\, \nu$ transitions
* main obstacle is the large background from $b \to c\, l\, \nu$ decays, whose elimination requires tight experimental cuts that are difficult to account for theoretically

  $\Rightarrow$ there is no single best way of measuring $|V_{ub}|$
* study of exclusive and inclusive $B$ decays allows extractions of $|V_{ub}|$ with very different kinds of theoretical uncertainties

# $|V_{ub}|$ *from Exclusive Decays*

$*$ theory must provide predictions for form factors with controlled theoretical errors

- least sophisticated approach
- $\delta|V_{ub}|/|V_{ub}| = \delta F(q^2)/F(q^2)$ at any value of $q^2$

*Different methods:*

- lattice QCD
- light-cone QCD sum rules
- other techniques
- flavor symmetries

## *Form Factors from Lattice QCD*

∗ method of choice for a precise determination free of uncontrolled errors

∗ recent progress in calculation of $B \to D^{(*)}$ form factors very encouraging
[→ A. Kronfeld]

∗ but heavy-to-light transitions pose new challenges:

• for $B \to \pi$ physics near zero recoil is affected by $B^*$ pole contribution ($B^*$–$B$ mass splitting is off by about a factor 2 in the quenched approximation)

• for $B \to \rho$ need to get $\rho$ lineshape right (problem of unstable particles in euclidean space)

• calculation of chiral corrections (lattice–continuum matching) has its intricacies

∗ on a timescale of 5 years, none of these appear as a show stopper

## *Form Factors from QCD Sum Rules*

* next best theoretical tool are light-cone QCD sum rules, which are specifically designed for describing heavy-to-light transitions near zero recoil
* in past years, significant refinements have been achieved (NLO corrections, higher-twist effects, target-mass corrections)
    - although not entirely model-independent, sum rules make a large number of testable predicitons, which can be confronted with data
    - state-of-the-art predictions can be trusted at level of 10–20%
* matching between lattice and sum rule calculations in some range of $q^2$ values provides a powerful consistency check of both techniques

## *Other Techniques*

* soft-collinear effective theory implies novel relations between heavy-to-light form factors in the heavy-quark limit

  • have been implemented in relativistic quark models, which are much more sophisticated than earlier models such as BSW, KS, ISGW

* constraints on shapes of heavy-to-light form factors can be derived using analyticity and unitarity (dispersive approach)

* promising strategy is to combine information from lattice, sum rules, heavy-quark relations, and dispersive methods [→ Postler]

## *Form Factors using Flavor Symmetries*

* idea is to use a combination of heavy-quark and SU(3) symmetries to derive relations between various $B \to \pi, \rho$ and $D \to \pi, K, K^*$ form factors at the same value of recoil velocity
* using charm data, the $B \to \pi, \rho$ form factors can then be predicted up to symmetry-breaking corrections
* nice academic excercise, but reliance on $\Lambda/m_c$ expansion and SU(3) symmetry is a severe drawback
  - both $\Lambda/m_c$ and SU(3) breaking are expected to be large in the charm sector (don't trust $20\% \times 20\% = 4\%$ arithmetics!)
  - not clear that a "recoil window" exists where the symmetry relations work (limited kinematic range in $D$ decays)
* precision less than for any decent quark model (most models respect symmetry relations, and so spread between models is an estimator of theoretical errors)

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* answer not clear!
  - trade-off between Fermi motion effects and duality violations and/or larger power corrections
  - universality of shape function in $B$ decays into massless partons implies many powerful relations

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*Would you avoid using parton distribution functions in DIS and collider physics?*

*Some prominent strategies:*

∗ cut on charged-lepton energy: $E_l > \frac{m_B^2 - m_D^2}{2m_B}$

$\Rightarrow$ eliminates $\sim 90\%$ of signal events

∗ cut on invariant hadronic mass: $M_H < m_D$

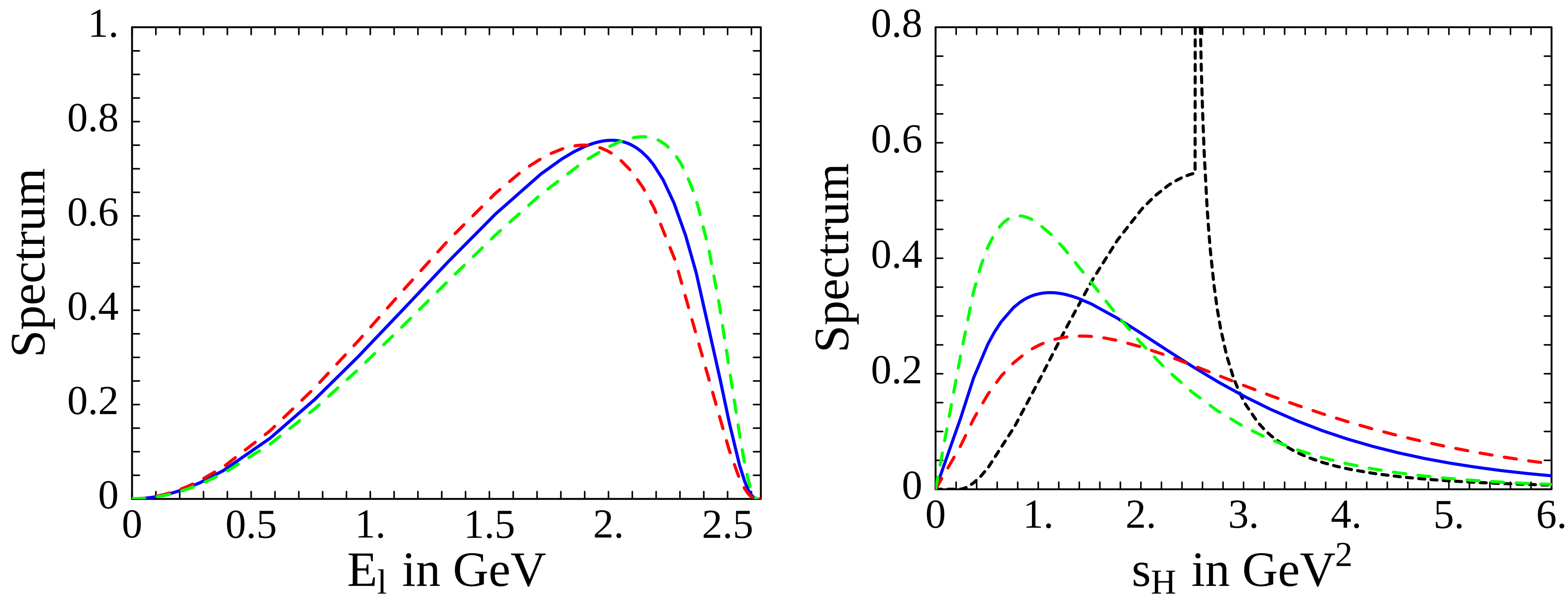
$\Rightarrow$ eliminates $\sim 20\%$ of signal events

∗ cut on invariant leptonic mass: $M_{l\nu} > m_B - m_D$

$\Rightarrow$ eliminates $\sim 80\%$ of signal events

∗ for last strategy, characteristic scale of heavy-quark expansion $\mu_c \approx 1$ GeV is much less than $b$-quark mass

* for first two strategies, results are sensitive to Fermi motion of the $b$ quark ("QCD smearing" of parton spectra, described by a nonperturbative shape function



## $|V_{ub}|$ *using Radiative* $B$ *Decays*

* at leading power in $\Lambda/m_b$, Fermi motion effects can be eliminated by taking a ratio of weighted integrals over the endpoint regions of the charged-lepton energy spectrum in $B \to X_u\, l\, \nu$ and the photon energy spectrum in $B \to X_s\gamma$: [M.N., 1993; $\to$ Rothstein]

$$\left|\frac{V_{ub}}{V_{cb}}\right|^2 \simeq \left|\frac{V_{ub}}{V_{tb}V_{ts}^*}\right|^2 = \frac{3\alpha}{\pi}\, K_{\rm pert}\, \frac{\widehat{\Gamma}_u(E_0)}{\widehat{\Gamma}_s(E_0)} \;+\; O\Big(\frac{\Lambda}{m_B}\Big)$$

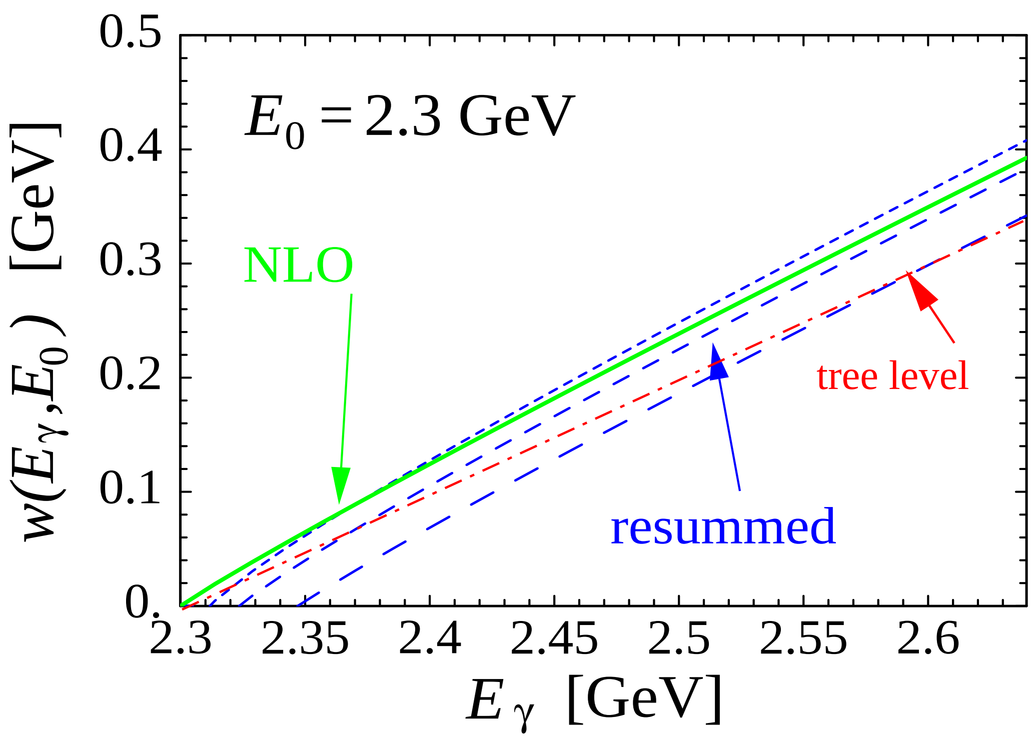
where:

$$\widehat{\Gamma}_u(E_0) = \int_{E_0}^{m_B/2} dE_l\, \frac{d\Gamma(B \to X_u\, l\, \nu)}{dE_l}$$

$$\widehat{\Gamma}_s(E_0) = \frac{2}{m_B} \int_{E_0}^{m_B/2} dE_\gamma\, w(E_\gamma, E_0)\, \frac{d\Gamma(B \to X_s\gamma)}{dE_\gamma}$$

* short-distance correction $K_{\rm pert}$ and weight function $w(E_\gamma, E_0)$ known at NLO
* results are stable:

$$K_{\rm pert} = 0.134\,^{+0.007}_{-0.009}\,[\text{scale}]\,^{+0.007}_{-0.006}\,[m_c] \pm 0.010\,[\alpha_s^2]$$



* theoretically mature, little room for improvements
* results allow determination of $|V_{ub}|$ with decent theoretical uncertainty:

  $\delta|V_{ub}| = 5\%$ [pert.] $\oplus$ 10–20% $[\Lambda/m_b]$

*Main advantage:*

- Fermi motion effects cancel (at leading order)

*Main limitations:*

- duality violations (strong weighting towards resonance region)
- $\Lambda/m_b$ corrections difficult to control

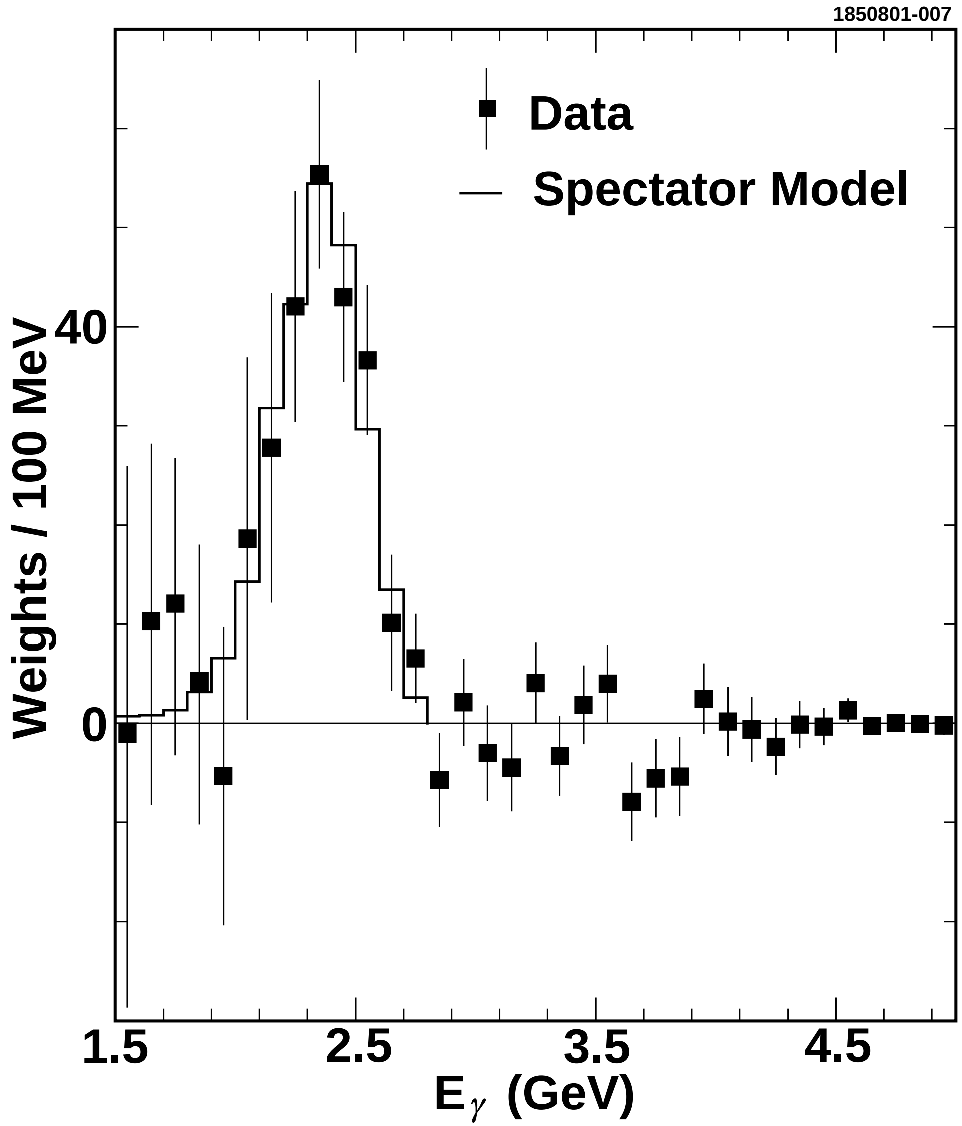
* importance of these effects can be probed by checking that result for $|V_{ub}|$ is independent of the experimental cutoff $E_0$ employed

## $|V_{ub}|$ via Extraction of the Shape Funtion

* shape function can be determined with good precision from $B \to X_s\gamma$ photon spectrum (requires a deconvolution, but experimenters do that all the time)
* facilitated by fact that parton spectrum is, to a good approximation, a spike: $d\Gamma/dE_\gamma \propto \delta(E_\gamma - m_b/2)$

  (this fact is also used in the global $V_{cb}$, $\bar{\Lambda}$, $\lambda_1$ fit [$\rightarrow$ Becher])

∗ here is the shape function (almost...) measured by CLEO:



* once shape function is known, all $B \to X_u \, l \, \nu$ decay distributions can be predicted over their entire kinematic range

- allows one to use $E_l$, $M_H$, $E_H$, $M_{l\nu}$ spectra or combinations thereof
- analog of what is routinely done in DIS and collider physics
- shape function parameterization can be refined as the data get more precise $\Rightarrow$ no intrinsic limitation

*Advantages:*

- optimal use of data
- least susceptible to duality violations
- first-order $\Lambda/m_b$ correctins switch themselves off as kinematic regions are enlarged

*Limitation:*

- mostly experimental (quality of $B \to X_s \gamma$ data)

## *Avoiding the Shape Function*

* sensitivity to Fermi motion effects can be eliminated by cutting out the "collinear region" $E_H \gg M_H$

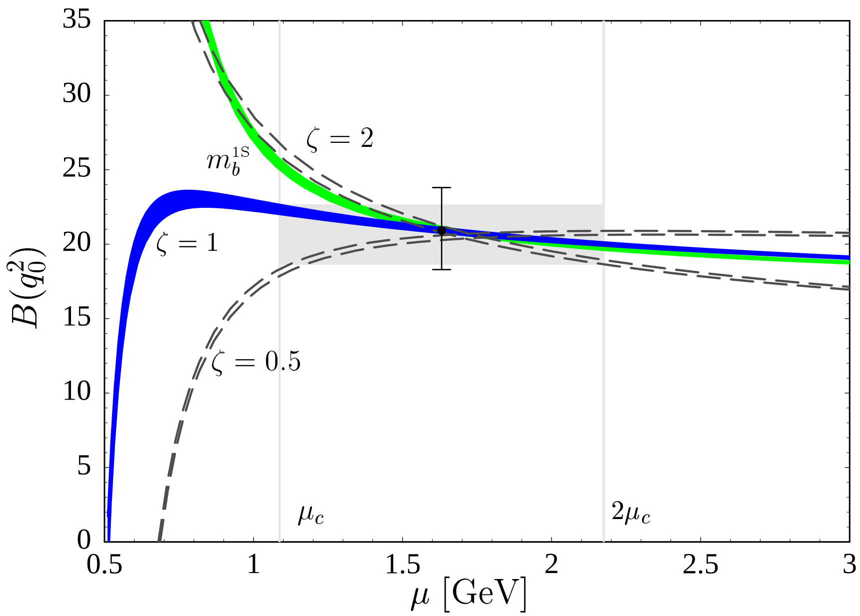
  • experimental cut $E_H < m_D$ converts "shape function terms" $\Lambda E_H / M_H^2 \sim 1$ into power corrections $\sim (\Lambda/\mu_c)$

  • formally, this yields a local OPE in powers of $\Lambda/\mu_c$

  • some technical complexities arise from the simultaneous expansion in small ratios $\mu_c/m_b$ and $\Lambda_{\rm QCD}/\mu_c$ ("hybrid expansion")

* define, with $q_0^2 = (m_B - m_D)^2$:

$$\text{Br}(B \to X_u\, l\, \nu)|_{q^2 > q_0^2} \equiv B(q_0^2) \times |V_{ub}|^2 \left( \frac{\tau_B}{1.6\,\text{ps}} \right)$$




* result:

$$B(q_0^2) = 20.9\,^{+2.9}_{-2.6}\,[m_b]\,^{+1.8}_{-2.3}\,[\text{pert.}] \pm 2.0\,[(\Lambda/\mu_c)^3] = 20.9 \pm 4.0$$

$\Rightarrow$ allows determination of $|V_{ub}|$ with $\sim 10\%$ accuracy

*Main advantage:*

- formally a local OPE, controlled by expansion in powers of $\Lambda/m_c$

*Main limitations:*

- ditto!
- dominant source of error is the guestimate of $(\Lambda/m_c)^3$ corrections (known to be huge for charm lifetimes)
- with $\Lambda = 0.5\,\text{GeV}$ these have a 5% effect on $|V_{ub}|$; with $\Lambda = 0.7\,\text{GeV}$ they have a 15% effect; with $\Lambda = 1\,\text{GeV}$ they have an 40% effect
- will be difficult to reduce or eliminate this uncertainty

*Main advantage:*

- formally a local OPE, controlled by expansion in powers of $\Lambda/m_c$

*Main limitations:*

- ditto!
- dominant source of error is the guestimate of $(\Lambda/m_c)^3$ corrections (known to be huge for charm lifetimes)
- with $\Lambda = 0.5\,\text{GeV}$ these have a 10% effect on $|V_{ub}|$; with $\Lambda = 0.7\,\text{GeV}$ they have a 30% effect; with $\Lambda = 1\,\text{GeV}$ they have an 80% effect
- hard to see how this uncertainty could be reduced or eliminated

$*$ to paraphrase Zoltan Ligeti (HEP 2001):

> *Unfortunately, it is much easier to write down a $\Lambda/m_c$ expansion than to test its convergence!*

## *Combination of Different Cuts*

* combinations such as $M_{l\nu}$ cut $\oplus$ $M_H$ cut appear very promising [$\rightarrow$ Bauer, Menke]

  • Caveat: need to include $O(\alpha_s)$ corrections to $d\Gamma/dM_H$ spectrum for a realistic study of shape function effects

* typically, a trade-off between Fermi motion effects and duality violations and/or large $\Lambda/m_c$ corrections

* optimization should take into account both theoretical and experimental realities

* ultimately, such combinations are the best bet to reducing the error on $|V_{ub}|$ below the 10% level

# Summary

* clean determinations of $|V_{ub}|$ are difficult and hindered by the necessity to eliminate the large charm background

* variety of proposals exist, each with their own theoretical limitations

* combination of various methods will help to reduce the error on $|V_{ub}|$ below the $10\%$ level and get confidence in the reliability of the theoretical methods