Extracting V_{ub} From Radiative Decays

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Introduction

- Extract V_{ub} in a Model Independent Fashion
  - In Principle: total charmless rate $\Gamma \propto m_b^5 G_F^2 | V_{ub} |^2$
  - Assumption: Inclusive rate allows for equating partonic and hadronic rate (Bloom-Gilman duality)
  - Justification: HQET + OPE

$$\Gamma_B = \frac{m_b^5 G_F^2}{96\pi^3} \mid V_{ub} \mid^2 \left(C_1(\alpha_s) + \frac{\lambda_1}{2m_b^2} - \frac{9\lambda_2}{2m_b^2} + O(\frac{\Lambda^3}{m_b^3}) \right)$$

- For the totally inclusive rate have very mild assumption of local duality, expected to be negligible. “Almost” on same footing as D.I.S. More on errors later.
- Too good to be true
- The need to implement the cut to remove overwhelming charmed background makes life difficult on the theory

The Lepton Energy Cut

- $E_L > (m_B^2 - m_D^2)/(2m_B)$
- “Easy” for Experiment, no neutrino reconstruction
- However, probes the end-points spectrum.

$$(m_B^2 - m_D^2)/(2m_B) \approx 0.87 \leq x_B \leq 1 \qquad x_B \equiv \frac{2E_l}{m_B}$$

The Problem of Fermi Motion

$$x_{parton}^{max} = 1 - \frac{\bar{\Lambda}}{m_B} \approx 0.9$$

- So Clearly soft hadronic *incalculable* physics plays a dominant role in the end point spectrum.
- Fermi motion of heavy quark accounts for difference in partonic and hadronic endpoints.
- Old days model Fermi motion

Modern Approach

- Can show from first principles that

$$\frac{d\Gamma}{dE} = \int_{2E-m_b}^{\bar{\Lambda}} dk_+ f(k_+) \frac{d\Gamma}{dE}(m_b^*) + O(\frac{\Lambda}{m_b}); (m_b^* = m_b + k+)$$

$$\int e^{-iyk_+} f(k_+) = \langle B(v) \mid \bar{b}_v(0) e^{i\int_0^y A\cdot d\lambda} b_v(y) \mid B(v) \rangle$$

- $f(k_+)$ is *universal* up to power corrections.
- In principle measurable on the lattice.
- Fortunately, $f(k_+)$ also dominates the end point of radiative decays.

Eliminate Structure Function

- Go to moments space and take ratio of rates.

$$|\frac{V_{ub}}{V_{cb}}|^2 = \frac{3\alpha}{\pi} C_7^2 (1+H^{\gamma}_{mix}) \int_{X_B^c}^1 dx_B \frac{d\Gamma}{dx_B} \times \left(\int_{x_B^c}^1 du_B W(u_B) \frac{d\Gamma^{\gamma}}{du_B} \right)^{-1}$$

$$W[u_B] = u_B^2 \int_{x_B^c}^{u_B} dx_B \qquad \Big(1 - 3(1-x_B)^2 + \frac{\alpha_s}{\pi}\big(\frac{7}{2} - \frac{2\pi^2}{9} - \frac{10}{9} Log(1 - \frac{x_B}{u_B})\big)\Big) .$$

- Accurate to $O(\alpha_s(1-x), (1-x)^3, \frac{\Lambda}{m_b})$

- Using the data, we find that including the non-trivial energy dependent radiative correction shifts the result by $\%35$. Though the energy independent correction from H_{mix} is order $\%50$ correction, due to relatively large size of C_2.
- Large $log(1-x_c)$ corrections as the cut approaches 1.
- Not clear whether these logs jeopardize expansion in α_s.

The end-point log problem

- Physically arise as a consequence of the fact that the probability NOT to radiate vanishes.
- On very general grounds it is possible to show that the form for the ratio of differential decay rates dR/dx is:

$$
\begin{aligned}
dR/dx \;\; \propto \;\; & \alpha_s Log(\epsilon) + \\
& \alpha_s^2 Log^2(\epsilon) + \alpha_s^2 Log(\epsilon) + \\
& \alpha_s^3 Log^3(\epsilon) + \alpha_s^3 Log^2(\epsilon) + ...
\end{aligned}
$$

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$\epsilon \equiv 1 - x_c$

The end-point log problem

- The series can be resummed into the form

$$dR/dx \propto Exp[F(\alpha_s Log(\epsilon)) + \alpha_s G(\alpha_s Log(\epsilon)) + ...]$$

- The net effect of the resummation is to change $W(u_B)$

$$W(u_B) = u_B^2 \int_{x_B^c}^{u_B} dx_B K \left[x_B; \frac{4}{3\pi\beta_0} Log(1 - \alpha_s \beta_0 L_{x/u}) \right]$$

$$L_{x/u} \equiv Log\left(-Log(x_B/u_B)\right)$$

- Including resummation changes result (using data) by $\%15$.

Conclusions: Lepton energy Cut

- Errors on order of $O(\alpha_s^2),\ O(\Lambda/m_b), O(\alpha_s(1-x_c))$
- "End-Point Log" problem is not a problem. If we wish we can inlclude effects of resummation of these logs. Will shift central value but wont effect error bars which are stil dominated by Λ/m_n.
- Only captures around $\%10-\%20$ of charmless events.
- How many resonances? Worry about duality errors.

Hadronic Mass Cut $m_x < m_D$

- Necessitates neutrino reconstruction
- Keeps approximately $\%80$ of total charmless events
- More support for larger invariant masses (more resonances)
- Expect duality to work reasonably well
- Again sensitive to Fermi motion as in lepton energy cut, i.e. sensitive to small mass high energy hadronic states.

Hadronic Mass Cut

- Again we can eliminate model dependence by utilizing the radiative decay data.

$$\frac{6\alpha C_7^2(1+H_{mix})\delta\Gamma(c)}{\pi(I_0(c)+I_+(c))} = \frac{\mid V_{ub}\mid^2}{\mid V_{ts}^* V_{tb}\mid^2}$$

- I_0, I_+ Weighted integrals over radiative decay data.
- End point logs negligible in this case.

Conclusions

- Can make model independent extractions of $\mid V_{ub} \mid$ with dominant error on the order of $O(\Lambda/m_b)$, by cutting on the leptonic energy or the hadronic mass at the price of utilizing the radiative decay data.
- Energy cut has the advantage that its easier experimentally, but has the disadvantage that it only captures about $\%10$ of the charmless rate. We worry about local duality violation.
- Mass cut necessitates neutrino reconstruction, but captures the largest (of all cuts) percentage ($(\%80)$) of the total charmless rate. Expect local duality to be more trustworthy assumption.

Conclusions

- Very tricky business, confidence will be gained once we see convergence of extractions. Local duality will always be lurking, but multiple extractions will teach us a lot. In particular given better resolution, vary the cut on the hadronic mass and see if extraction deviates.