CLEO's Analysis on Hadronic Mass Moments in Semileptonic B Decays

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V_{xb} Workshop
SLAC 12/07/01

Motivation and Method

- Measure V_{cb} with smallest experimental and theoretical errors

- Until recently two methods have been used to measure V_{cb}:
  - From measured exclusive $B \rightarrow D^{(*)} l \nu$ decay rate $d\Gamma/d\omega$ & HQET
    Get V_{cb} from extrapolation of $d\Gamma/d\omega$ to $\omega=1$ (zero recoil) $\Rightarrow F(1)\,|V_{cb}|$
    CLEO, ALEPH, DELPHI, L3, OPAL: $|V_{cb}| = (41.8 \pm 1.6_{exp} \pm 2.3_{th}) \times 10^{-3}$

  - From inclusive semileptonic branching fraction & HQE
    Get V_{cb} from predicted branching fraction
    ALEPH, DELPHI, OPAL: $|V_{cb}| = (40.7 \pm 0.45_{exp} \pm 2.03_{th}) \times 10^{-3}$

- New method: Measure moments of hadronic mass spectrum

$M1 \equiv \langle M_X^2 - \overline{M}_D^2 \rangle$, $M2 \equiv \langle (M_X^2 - \overline{M}_D^2)^2 \rangle$ and $M2' \equiv \langle (M_X^2 - \langle M_X^2 \rangle)^2 \rangle$

where $\overline{M}_D$ is spin averaged D meson mass $\overline{M}_D = \frac{1}{4} M_D + \frac{3}{4} M_{D^*} = 1.975$ GeV
and $\overline{M}_X$ mass of charmed hadron system X_c

Motivation

- Expand semileptonic decay rate to order $1/M_B^3$ and $\beta_0\alpha_s^2$ Using HQE:

$$\Gamma_{sl} = \frac{G_F |V_{cb}|^2 M_B^5}{192\pi^3} 0.3689[1 - 1.54\frac{\alpha_s}{\pi} - 1.43\beta_0\frac{\alpha_s^2}{\pi^2} + 1.648\frac{\bar{\Lambda}}{M_B}(1 - 0.87\frac{\alpha_s}{\pi}) - 0.946\frac{\bar{\Lambda}^2}{M_B^2} - 3.185\frac{\lambda_1}{M_B^2}$$
$$+ 0.02\frac{\lambda_2}{M_B^2} + 0.298\frac{\bar{\Lambda}^3}{M_B^3} + 3.28\frac{\bar{\Lambda}\lambda_1}{M_B^3} + 10.47\frac{\bar{\Lambda}\lambda_2}{M_B^3}$$
$$- 6.153\frac{\rho_1}{M_B^3} + 7.482\frac{\rho_2}{M_B^3} - 7.4\frac{T_1}{M_B^3} + 1.491\frac{T_2}{M_B^3} - 10.41\frac{T_3}{M_B^3} + 7.482\frac{T_4}{M_B^3} + O(1/M_B^4)]$$

- ➢ at $1/M_B$: one parameter $\bar{\Lambda}$ ⇒ energy of light quark & gluon do
  ➢ at $1/M^2_B$: two parameters λ_1 ⇒ average momentum-squared of b-quark
  λ_2 ⇒ energy of hyperfine interaction of S_b with $S_{light\ dof}$ ⇒ $\Delta m(B^*\text{-}B)$
  ➢ at $1/M^3_B$: 6 additional parameters: $\rho_1, \rho_2, T_1, T_2, T_3, T_4$ (fix these)

- Need to measure parameters $\bar{\Lambda}, \lambda_1, \lambda_2$

  Measure first and second moment of M_X in $B\to X_c l\nu$ and E_γ in $B\to X_s\gamma$

Hadronic Mass Moments

- **M1:** $$\frac{\langle M_X^2 - \bar{M}_X^2 \rangle}{\bar{M}_B^2} = [0.0272\frac{\alpha_s}{\pi} + 0.058\beta_0\frac{\alpha_s^2}{\pi^2} + 0.207\frac{\bar{\Lambda}}{\bar{M}_B}(1 + 0.43\frac{\alpha_s}{\pi}) + 0.193\frac{\bar{\Lambda}^2}{\bar{M}_B^2} + 1.38\frac{\lambda_1}{\bar{M}_B^2}$$
$$+0.203\frac{\lambda_2}{\bar{M}_B^2} + 0.19\frac{\bar{\Lambda}^3}{\bar{M}_B^3} + 3.2\frac{\bar{\Lambda}\lambda_1}{\bar{M}_B^3} + 1.4\frac{\bar{\Lambda}\lambda_2}{\bar{M}_B^3}$$
$$+4.3\frac{\rho_1}{\bar{M}_B^3} - 0.56\frac{\rho_2}{\bar{M}_B^3} + 2.0\frac{T_1}{\bar{M}_B^3} + 1.8\frac{T_2}{\bar{M}_B^3} + 1.7\frac{T_3}{\bar{M}_B^3} + 0.91\frac{T_4}{\bar{M}_B^3} + O(1/\bar{M}_B^4)$$
- **M2':** $$\frac{\langle (M_X^2 - \langle \bar{M}_X^2 \rangle)^2 \rangle}{\bar{M}_B^4} = [0.00148\frac{\alpha_s}{\pi} + 0.0025\beta_0\frac{\alpha_s^2}{\pi^2} + 0.027\frac{\bar{\Lambda}}{\bar{M}_B}\frac{\alpha_s}{\pi} + 0.0107\frac{\bar{\Lambda}^2}{\bar{M}_B^2} - 0.12\frac{\lambda_1}{\bar{M}_B^2}$$
$$+0.02\frac{\bar{\Lambda}^3}{\bar{M}_B^3} - 0.06\frac{\bar{\Lambda}\lambda_1}{\bar{M}_B^3} - 0.129\frac{\bar{\Lambda}\lambda_2}{\bar{M}_B^3}$$
$$-1.2\frac{\rho_1}{\bar{M}_B^3} - 0.0032\frac{\rho_2}{\bar{M}_B^3} - 0.12\frac{T_1}{\bar{M}_B^3} - 0.36\frac{T_2}{\bar{M}_B^3} + O(1/\bar{M}_B^4)$$
- **$\bar{M}_B$ = 5.313 GeV is spin-averaged B meson mass**
- **All except $\frac{\bar{\Lambda}}{\bar{M}_B}\frac{\alpha_s}{\pi}$ term are computed with $p_l > 1.5$ GeV restriction (good to 50%)**

Analysis Strategy

- Require one charged lepton (e or μ) with 1.5 GeV/c < p^*_l < 2.5 GeV/c
- Reconstruct ν: $E_\nu = E_b - \sum_{i=1}^{n} E_i$ and $\vec{p}_\nu = -\sum_{i=1}^{n} \vec{p}_i$
  Remove double counting between calorimeter and tracking measurements
- Require $M_\nu = \sqrt{E^2_\nu - p^2_\nu}$ consistent with zero; total charge in event = 0
- Use event shape requirements (R_2, thrust) to reduce $q\bar{q}$ backgrounds
- Sample composition (3.1 fb^{-1}):
  89% $B\bar{B}$ (95% $b \to c l \nu$; 2.8±0.6%secondary lepton, 2.1±1.1% $b \to u l \nu$)
  11% $q\bar{q}$
- Efficiency: 2%

Analysis Strategy

- **Determine hadronic mass system** $$M_X^2 = (E_B - E_l - E_\nu)^2 - (\vec{p}_B - \vec{p}_l - \vec{p}_\nu)^2$$ $$= M_B^2 + M_{l\nu}^2 - 2E_B E_{l\nu} + 2|\vec{p}_B||\vec{p}_{l\nu}|\cos\theta_{l\nu,B}$$
- **Since $\theta_{l\nu,B}$ is unknown, neglect last term and use approximation:** $$\tilde{M}_x^2 = M_B^2 + M_{l\nu}^2 - 2E_B E_{l\nu}$$
- **Expand moments in powers of $1/M_B$ up to $1/M_B^3$ and up to $\beta_0\alpha_s^2$**
- **2 Issues: i) Convergence of expansion**
  ii) Validity of assumptions of underlying expansion quark-hadron duality
- **Introduces additional uncertainty $\Rightarrow$ need to determine $\bar{\Lambda}$, λ_1, λ_2 with several methods**

Inputs

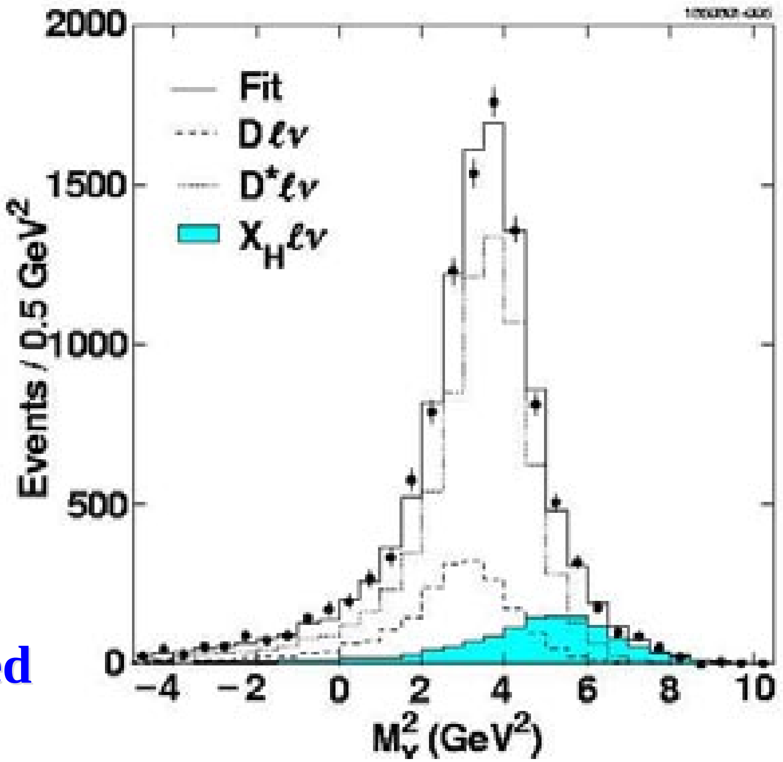
- $1/M^3_B$ parameters ρ_i and T_i are estimated from dimensional considerations $(0.5\ \mathrm{GeV})^3$

- Use $\rho_2 = 0\pm(0.5\ \mathrm{GeV})^3$
  $T_i = 0\pm(0.5\ \mathrm{GeV})^3$ i=1,4
  $\rho_1 = 1/2(0.5\mathrm{GeV})^3 \pm 1/2\ (0.5\ \mathrm{GeV})^3$

- Other inputs: $\lambda_2 = 0.128\pm0.019\mathrm{GeV}^2$ (appropriate with $O(1/M_B{}^3)$)
  $\alpha_s(m_b)=0.220$

M_X^2 Spectrum

- $\tilde{M}_X^2$ spectrum contains 11900 events after continuum background subtraction
- Spectrum is devided in 3 components $B \to D\, l\, \nu$, $B \to D^*\, l\, \nu$ & $B \to X_h\, l\, \nu$
- X_h is modeled with 6 resonances (ISGW2) $B \to D\pi\, l\, \nu$ & $B \to D^*\pi\, l\, \nu$ (Goity Roberts)
- $\langle M_X^2 \rangle = r_D M_D^2 + r_{D^*} M_{D^*}^2 + r_{X_h} \langle M_{X_h}^2 \rangle$, where r_D, r_{D*}, r_{X_h} are relative rates obtained from fit



- Measure: $$M1 \equiv \langle M_X^2 - \overline{M}_D^2 \rangle = 0.251 \pm 0.023 \pm 0.062\ \mathrm{GeV}^2$$
$$M2 \equiv \langle (M_X^2 - \overline{M}_D^2)^2 \rangle = 0.639 \pm 0.056 \pm 0.178\ \mathrm{GeV}^4$$
$$M2' \equiv \langle (M_X^2 - \langle M_X^2 \rangle)^2 \rangle = 0.576 \pm 0.048 \pm 0.163\ \mathrm{GeV}^4$$

smaller errors & correlations

Systematic Errors

- i) Largest systematic error from simulation of parameters that impact $\sigma_{m\nu}$:
  - ⇨ photon identification efficiency
  - ⇨ tracking efficiency
  - ⇨ additional neutrals: K_L, other ν's

  ➢ σ_{M1}^{sys} = 0.058 GeV^2
  ➢ σ_{M2}^{sys} = 0.140 GeV^4
  ➢ $\sigma_{M2'}^{sys}$ = 0.129 GeV^4

- ii) No knowledge about amount & shape of non-resonant contribution:
  - ⇨ fix fraction of non-resonant/resonant component in X_h in fit
  - ⇨ vary fraction systematically over limits that data allow

  ➢ σ_{M1}^{sys} = 0.011 GeV^2
  ➢ σ_{M2}^{sys} = 0.060GeV^4
  ➢ $\sigma_{M2'}^{sys}$ = 0.054 GeV^4

- iii) Other systematic errors are negligible:
  - ⇨ Subtraction of secondary lepton & $b \rightarrow ul\nu$
  - ⇨ Final state radiation correction
  - ⇨ Form factor uncertainties in $B \rightarrow D^{(*)} l \nu$

Theoretical Errors

- i) Determination of model dependence of high-mass region:
  - ⇨ Vary internal structure of resonance model by systematically dropping 6 contributing resonances, singly, in pairs and in triplets
    - ➢ $\sigma_{M1}^{th} = 0.015\ \mathrm{GeV}^2$
    - ➢ $\sigma_{M2}^{th} = 0.090\ \mathrm{GeV}^4$
    - ➢ $\sigma_{M2'}^{th} = 0.083\mathrm{GeV}^4$
- ii) Use phase space model for 2 non-resonant modes
  - ⇨ generates higher M_{Xh} than Goity-Roberts parameterization
  - ⇨ yield hadronic mass moments consistent with those in Goity-Roberts
  - ⇨ data constrains $r \bullet M^2$; individual components may vary significantly
- iii) Include scale uncertainty: $\alpha_s(m_b/2)= 0.275$ to $\alpha_s(2m_b)= 0.176$ and errors of ρ_1, ρ_2, T_1, …… T_4: $(0.5\ \mathrm{GeV})^3$
- Error correlation coefficients:

M1-M2:	$\pm 0.71_{stat}$	$\pm 0.5_{sys}$	$\pm 0.52_{tot}$
M1-M2':	$\pm 0.56_{stat}$	$\pm 0.34_{sys}$	$\pm 0.36_{tot}$

RESULTS

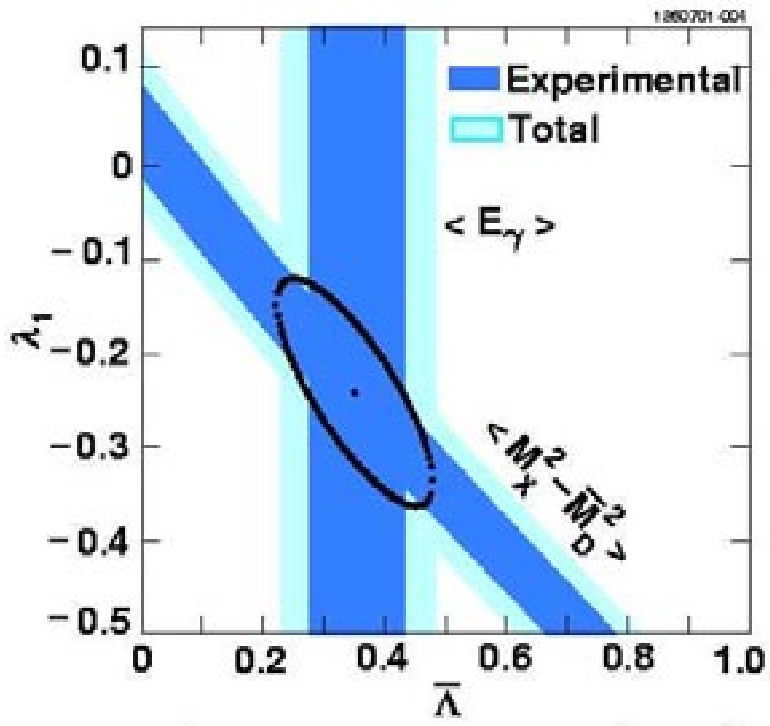
- Obtain band in $\bar{\Lambda}$-λ_1 plane from M1

- Use only M1 because of slow convergence of M2' in $1/M_B^3$

- CLEO determined moments E1 &E2 of E_γ spectrum in $b \to s\gamma$ to order $\beta_0\alpha_s^2$ & $1/M_B^3$

- Intersection of 2 bands yields:
  $\bar{\Lambda} = 0.35\pm0.07_{exp}\pm0.1_{th}$ GeV
  $\lambda_1 = -0.236\pm0.071_{exp}\pm0.078_{th}$ GeV

- Note: Results are scheme dependent ($\overline{MS}$) and order dependent



Results for V_{cb}

- CLEO measures: $B(B\to X_c l\nu)=10.39\pm0.46\%$
- Use lifetime averages $\tau_{B^+}=1.548\pm0.032$ ps & $\tau_{B^0}=1.653\pm0.028$ ps and CLEO measurement $f_{+-}/f_{00}=1.04\pm0.08$
  - $\Gamma_{sl} = 0.427\pm0.02\times10^{-10}$ MeV
- Extract $|V_{cb}|$ using measured values of Λ, λ_1 λ_2, Γ_{sl}, α_s, and assumptions for ρ_1, ρ_2, T_1, T_2, T_3, T_4
  - $|V_{cb}| =(4.04\pm0.09\pm0.05\pm0.08)\times10^{-2}$
- Note that theoretical error is σ_{th}:2% & experimental error is, σ_{exp}: 2.5%)
- This method gives smallest model dependence,