Goodness of Fit

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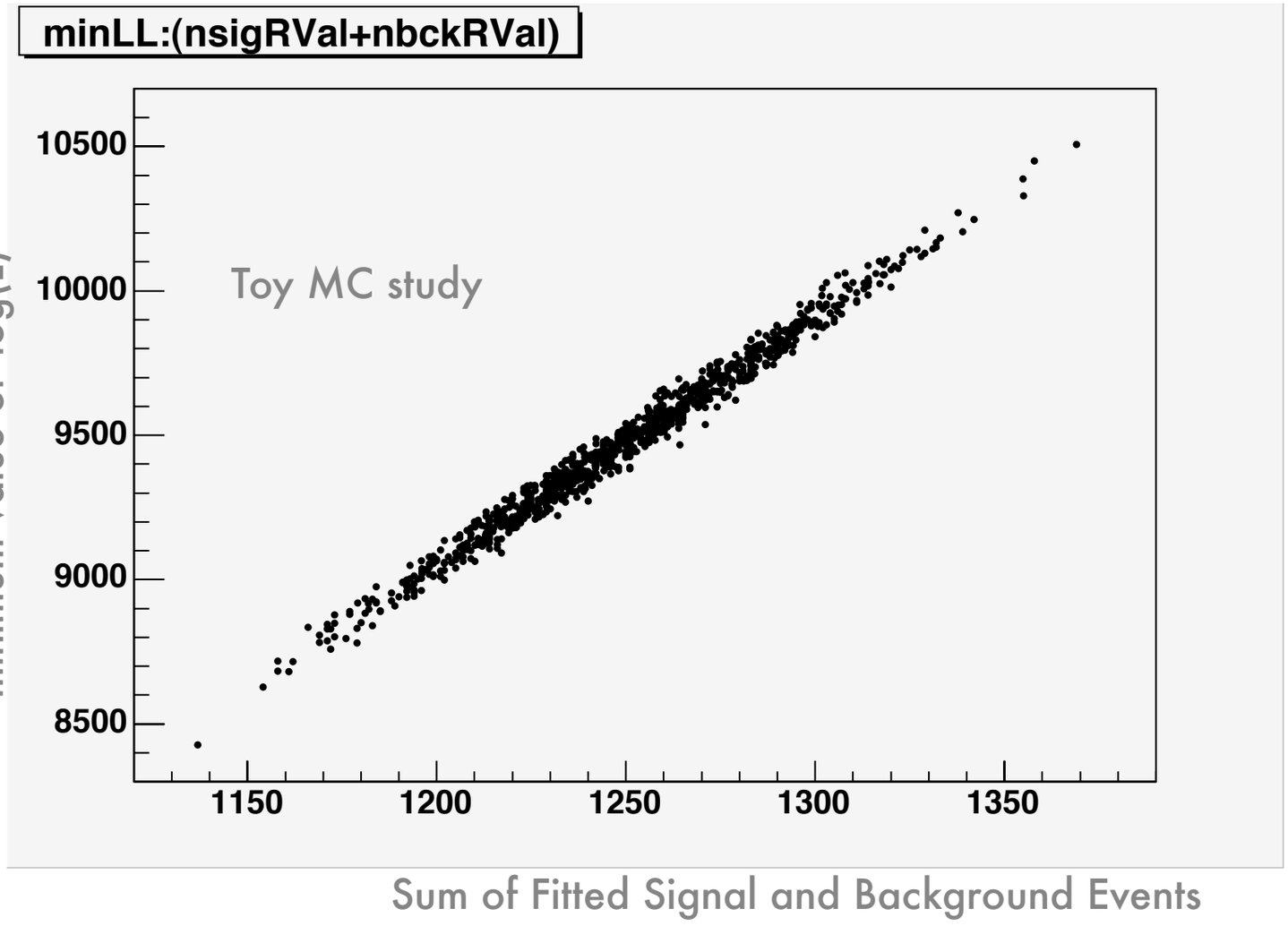
Goodness of Fit - Requirements

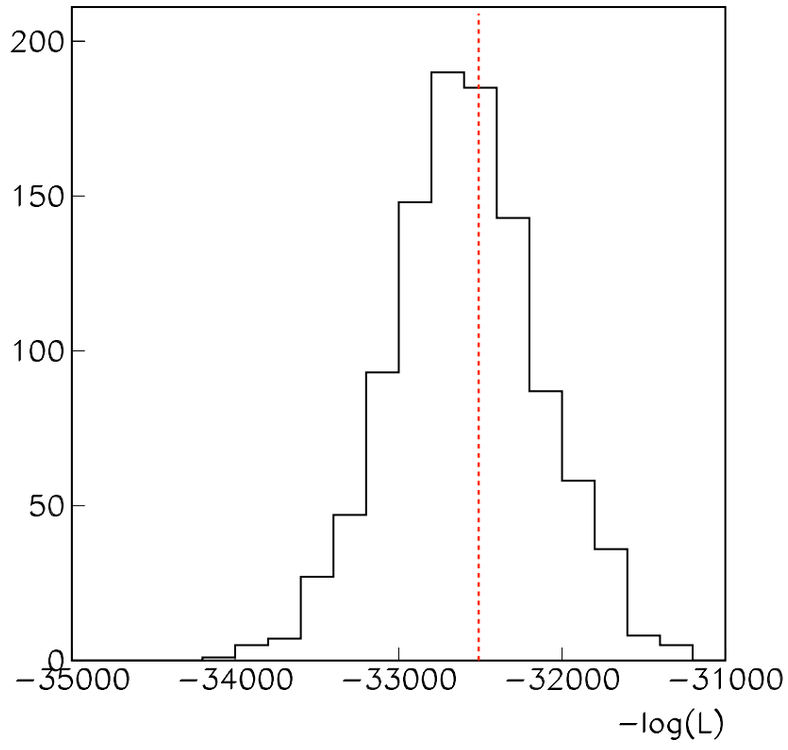
Statistic with sensitivity to the quality of the fit model

Should be Independent of Fitted parameters

have you ever wondered why all Babar min log(L) data vs Toy comparisons agree so well?

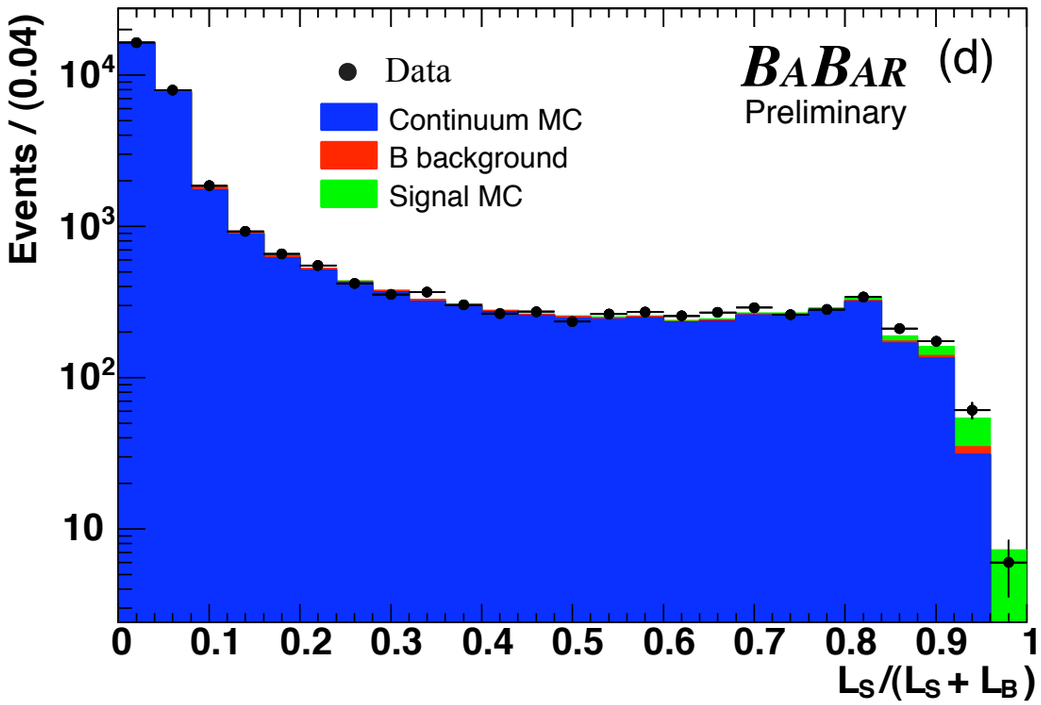


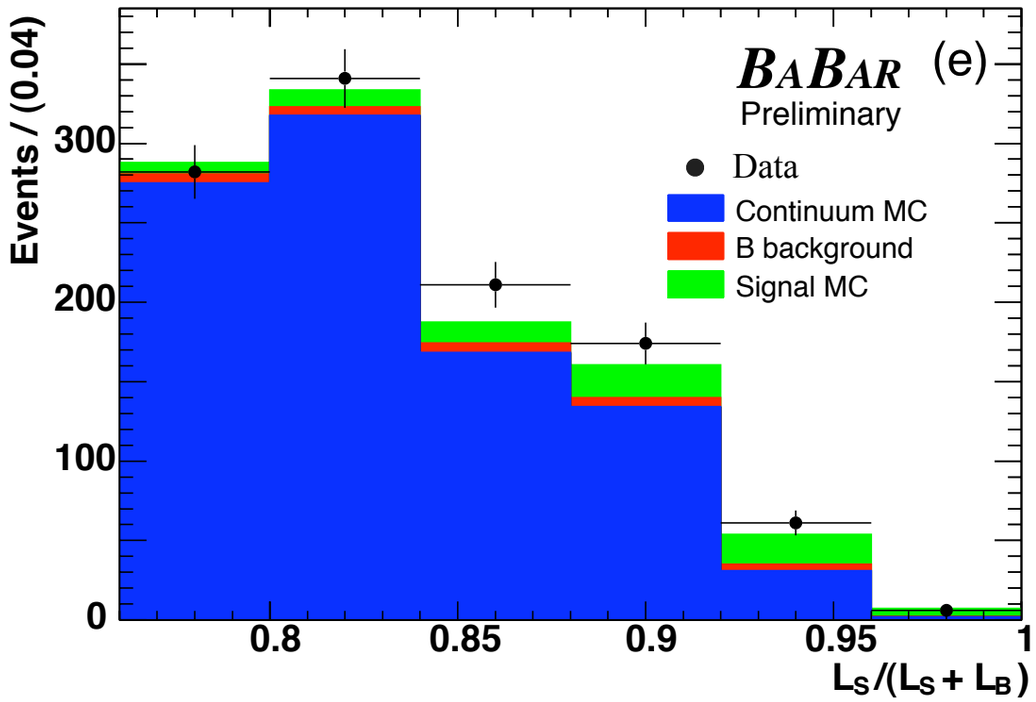
see Joel Heinrich's PHYSTAT2003 paper that demonstrates for simple PDFs that the log(L) is just proportional to a Fitted Parameter



PDF Ratio

$$\mathcal{R}_{PDF} = \frac{\mathcal{P}_{\text{signal}}(\vec{x};\vec{\alpha})}{\mathcal{P}_{\text{signal}}(\vec{x};\vec{\alpha}) + \mathcal{P}_{\text{background}}(\vec{x};\vec{\alpha})}$$





$$B^0 \to K^0_S \pi^0 \pi^0$$

Pdf Ratio is a 1-D projection of the full PDF

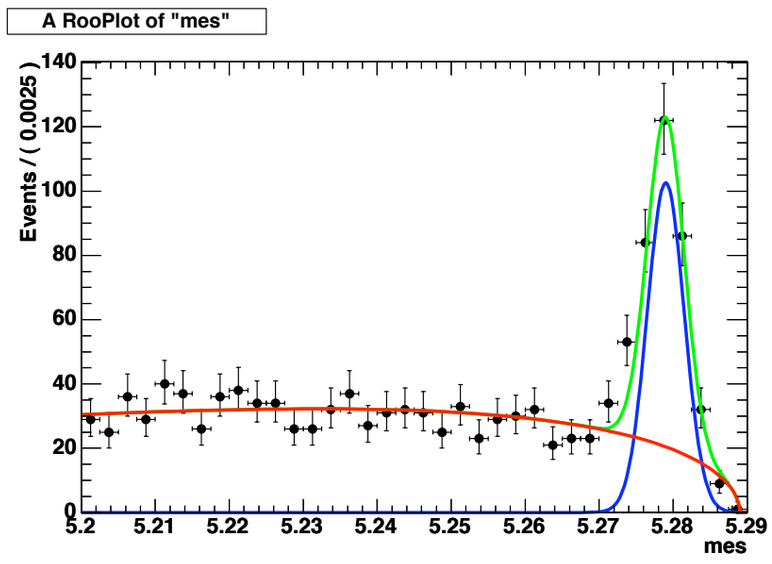
Is this Data vs. PDF comparison a Goodness of Fit test?

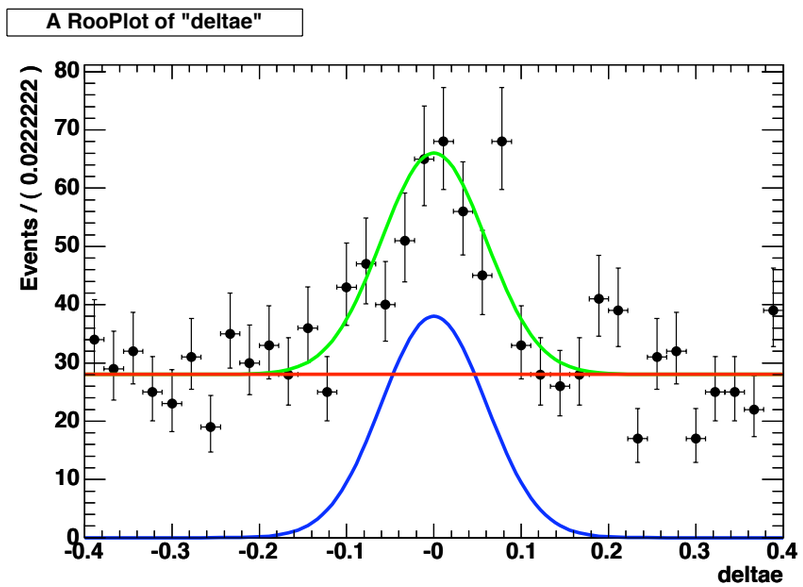
Toy MC Studies

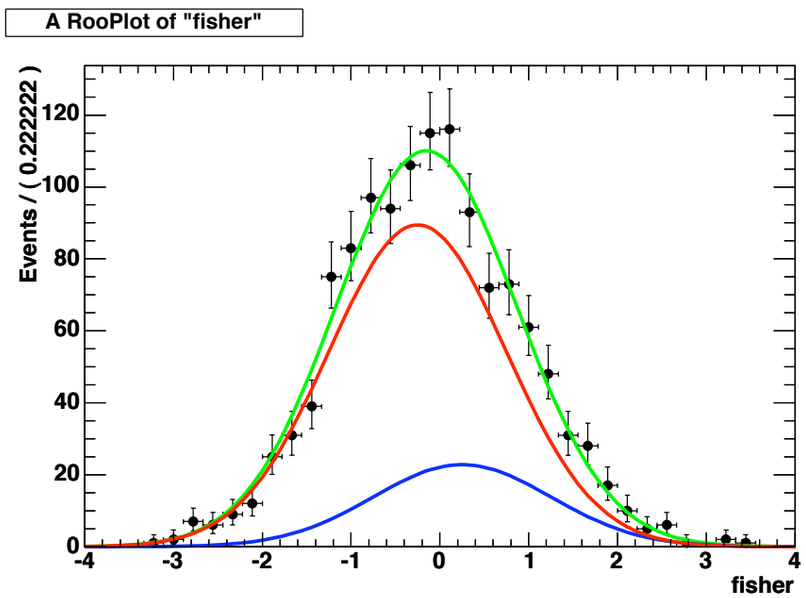
- 1-D and 3-D toy MC experiments
  - Generate with default PDF, or
  - Generate with a modified PDF, where we modify:
    - left-side $\sigma(m_{ES})$ in a Birfurcated Gaussian
    - $\sigma(\Delta E)$
    - the mean F value
  - Fit with the default PDF
- Goodness of Fit
  - compare PDF Ratio for Toy MC experiment and the Fitted PDF
  - use K-S Test distance to quantify comparison and find 90% efficient point for default PDF case
  - Power = fraction of Toy experiments with K-S Test distance > 90% point

Toy MC Experiments

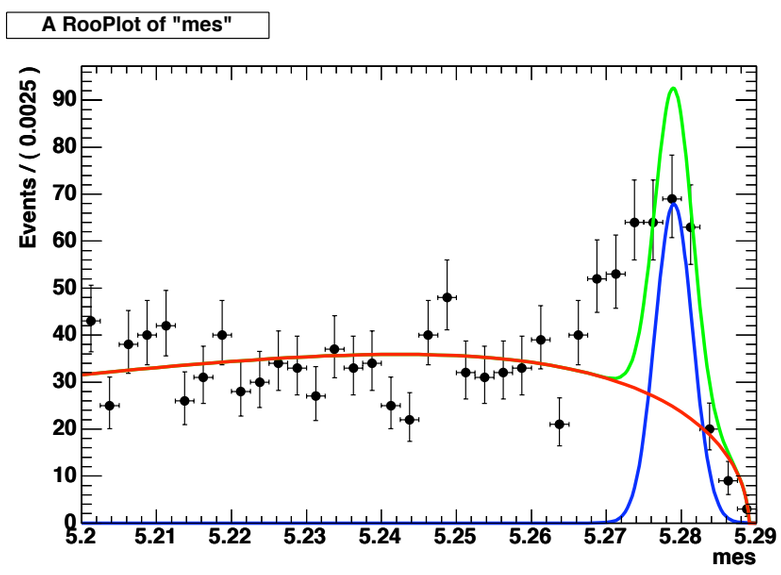
Toy MC generated with same PDF used to fit

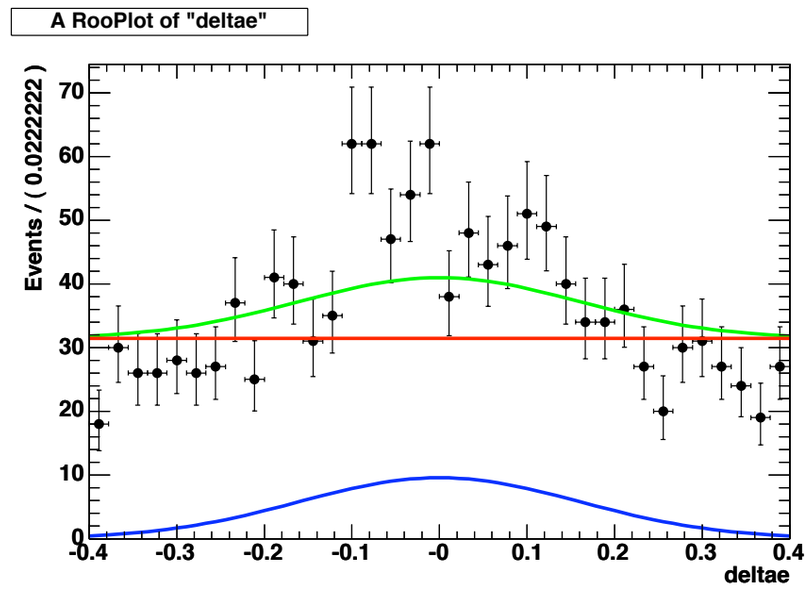


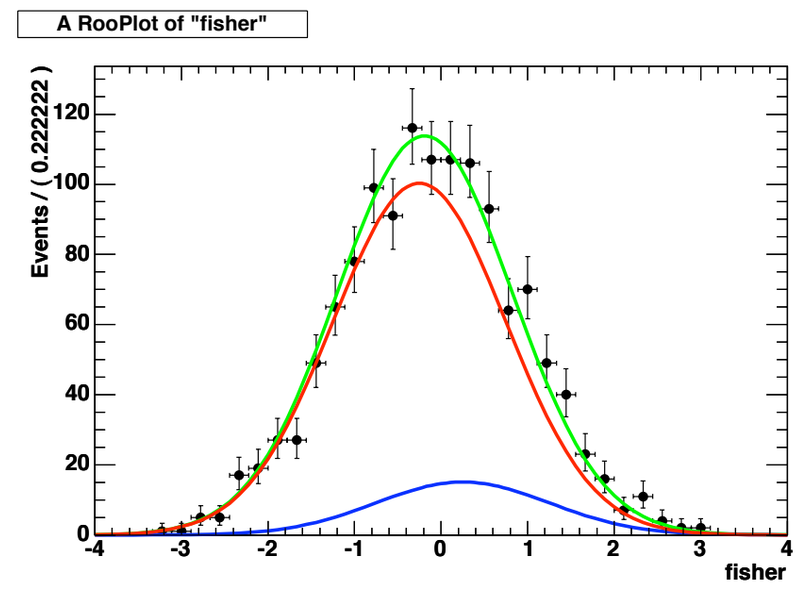




Toy MC generated with modified PDF



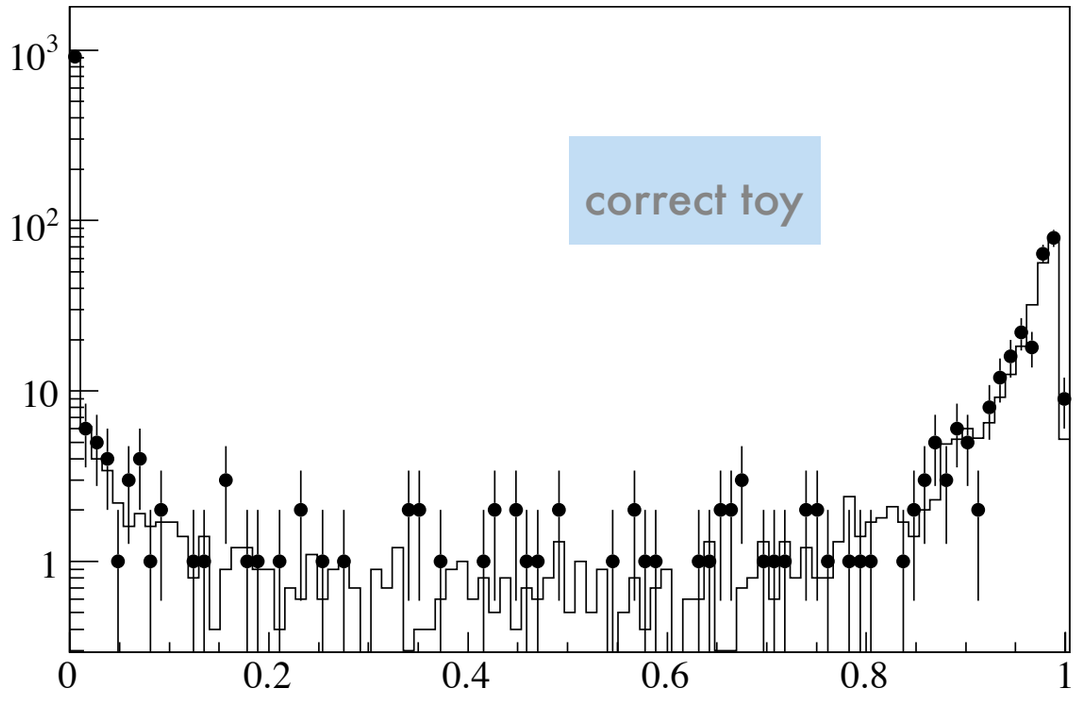


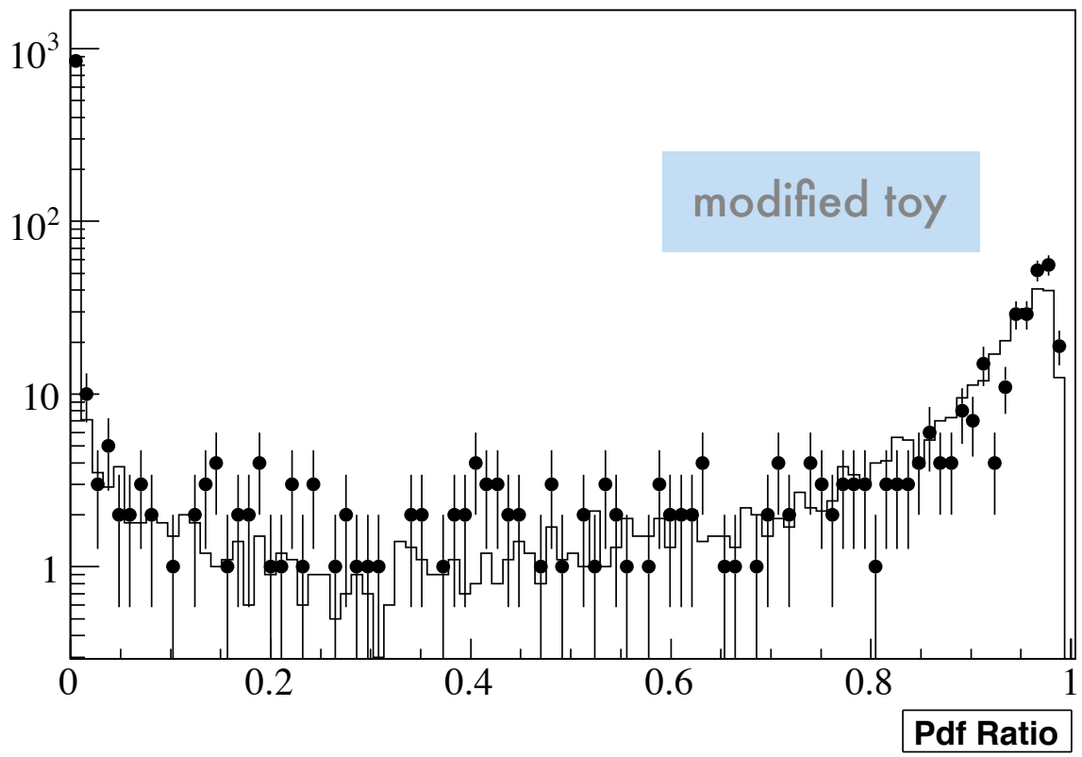


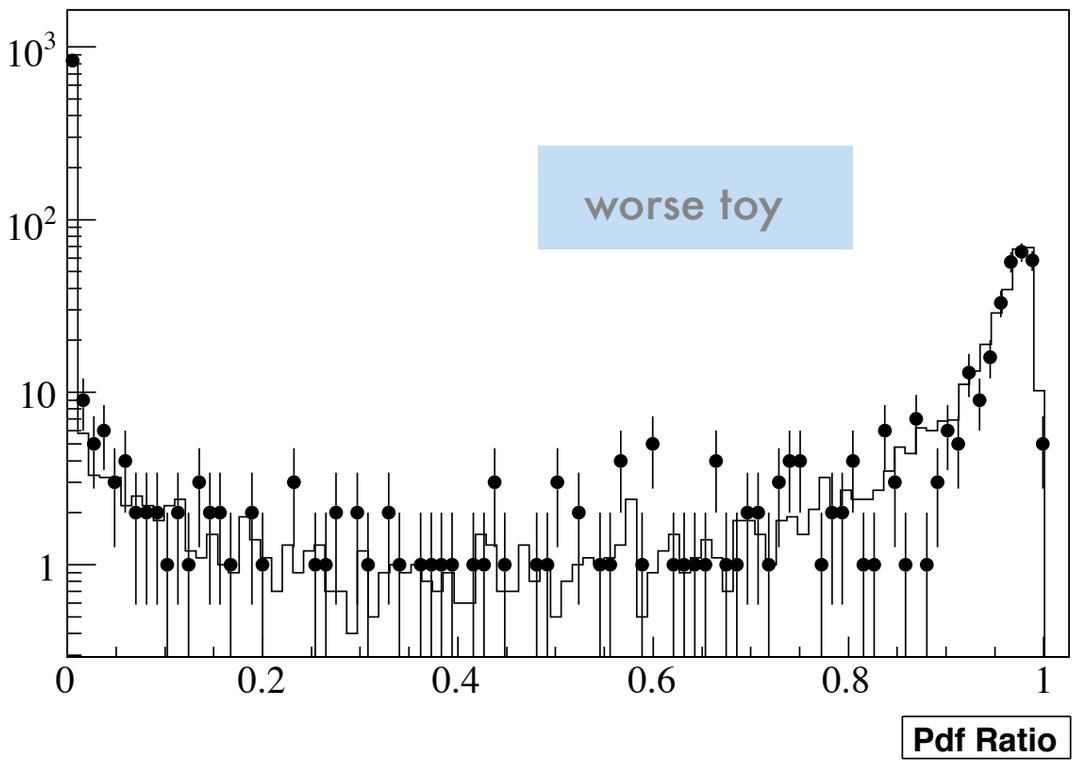
PDF Ratio Test on Toy MC

$$\mathcal{R}_{PDF} = \frac{\mathcal{P}_{\text{signal}}(\vec{x};\vec{\alpha})}{\mathcal{P}_{\text{signal}}(\vec{x};\vec{\alpha}) + \mathcal{P}_{\text{background}}(\vec{x};\vec{\alpha})}$$

Use the PDF with the parameters found from fitting the data





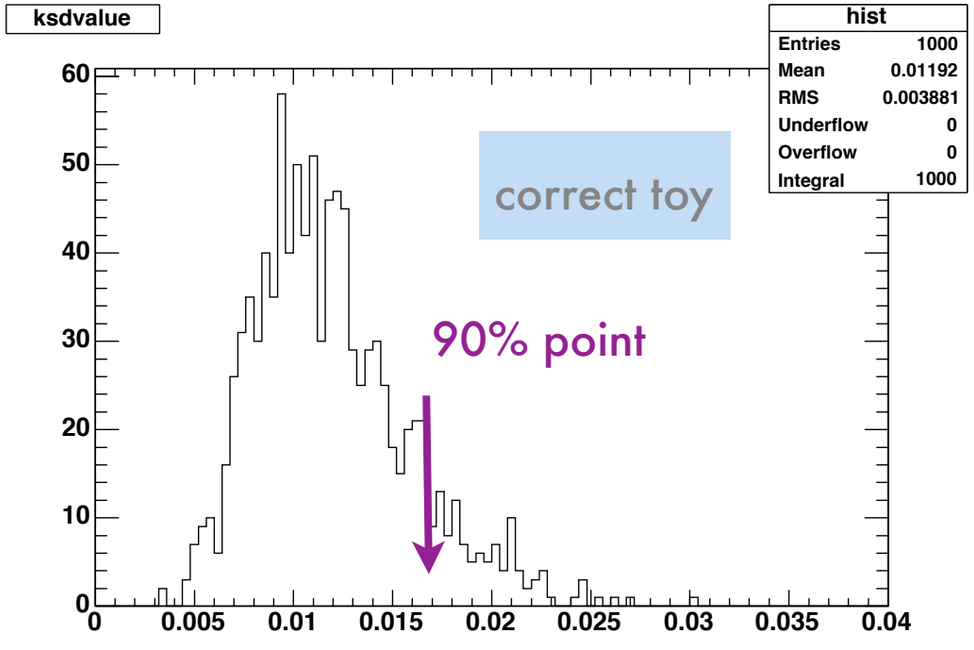


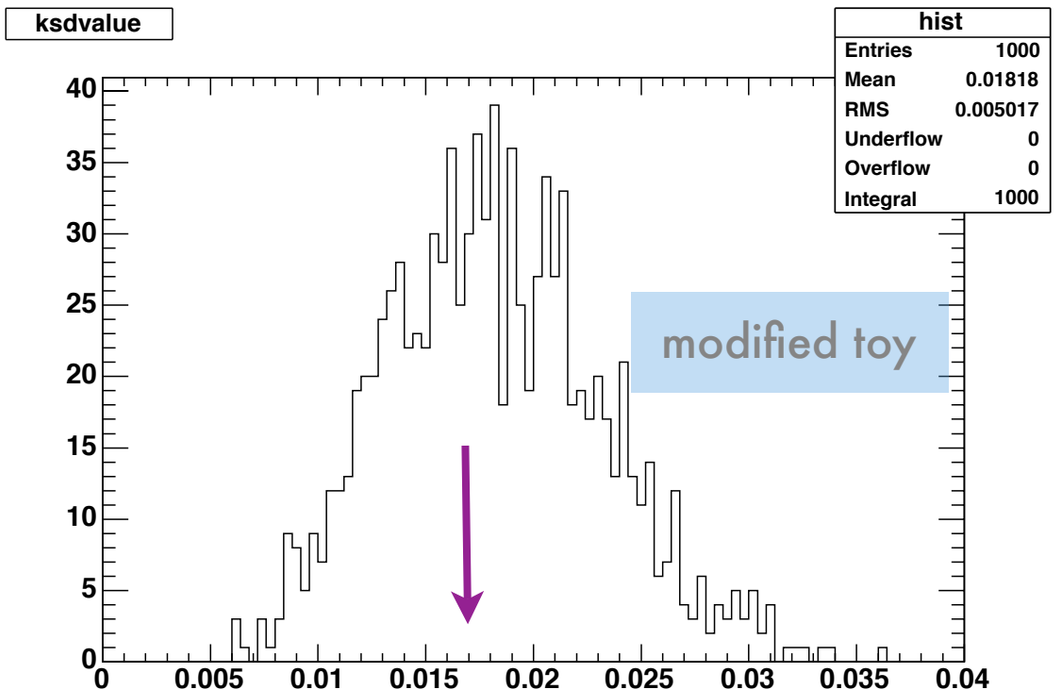
K-S Test Distance

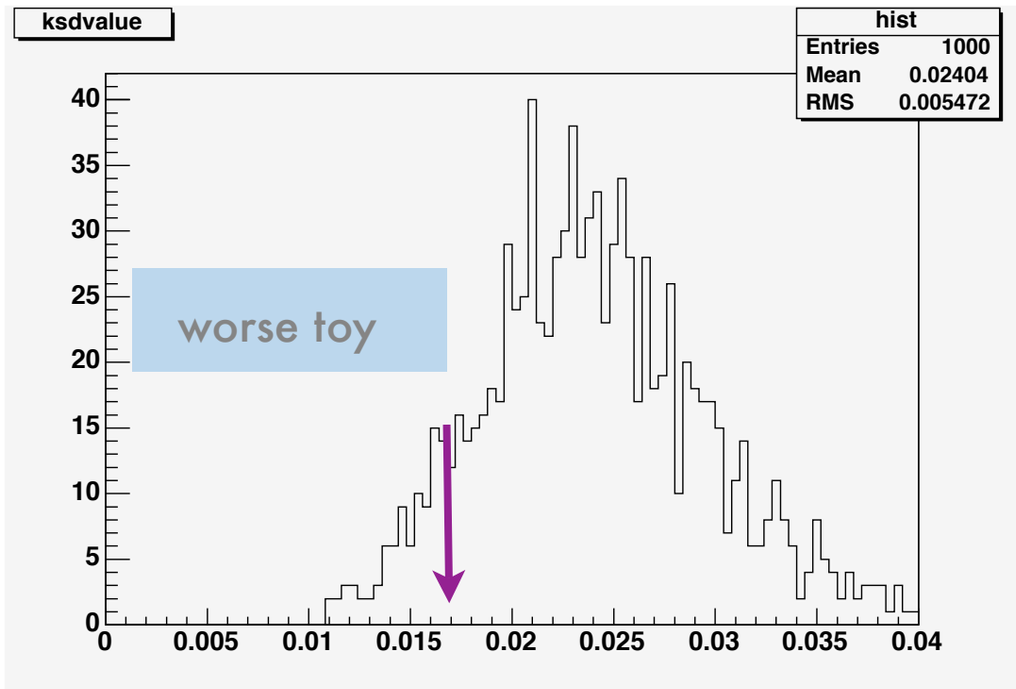
$$D_{K-S} = max \left| \int_{-\infty}^{t} \text{data}(R)dR - \int_{-\infty}^{t} \text{pdf}(R)dR \right|$$

compare toy MC generated with default PDF vs. 2 toys with modified PDFs

Goodness of Fit power improves as Fit PDF becomes worse





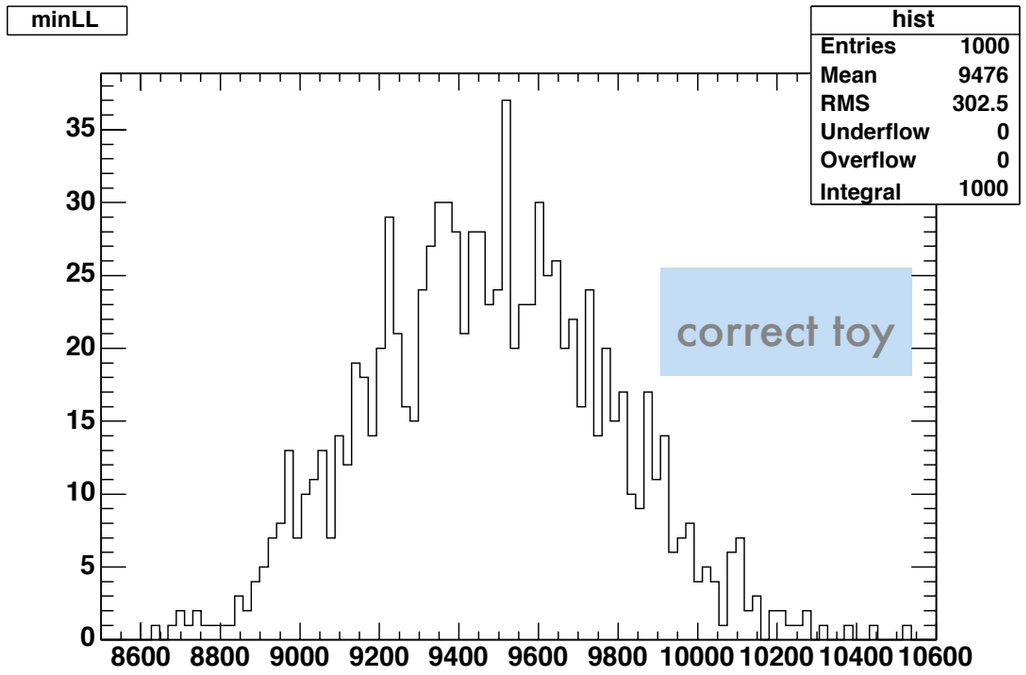


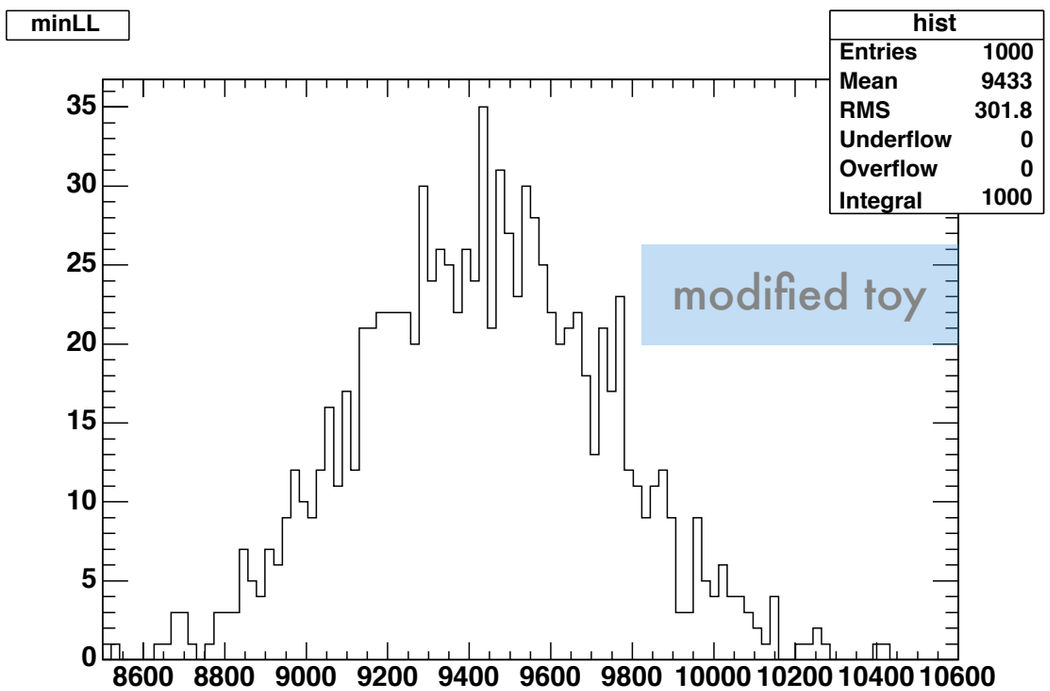
Value of -log(L)

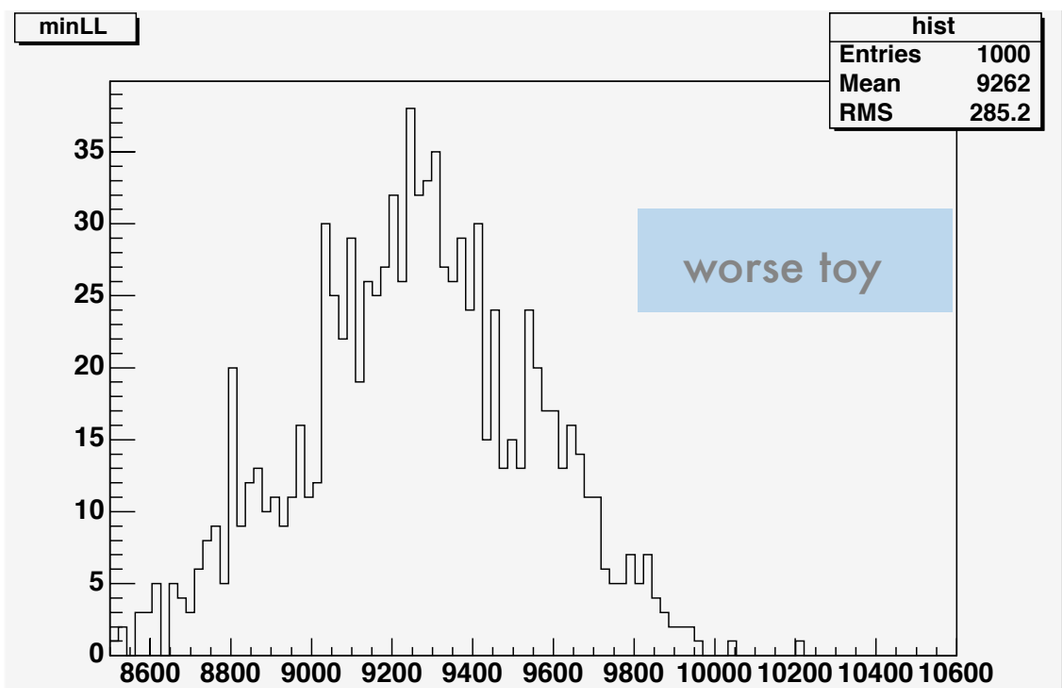
compare value of
-log(L) for 3 cases

no significant difference
in minimum -log(L) value for very poor fits

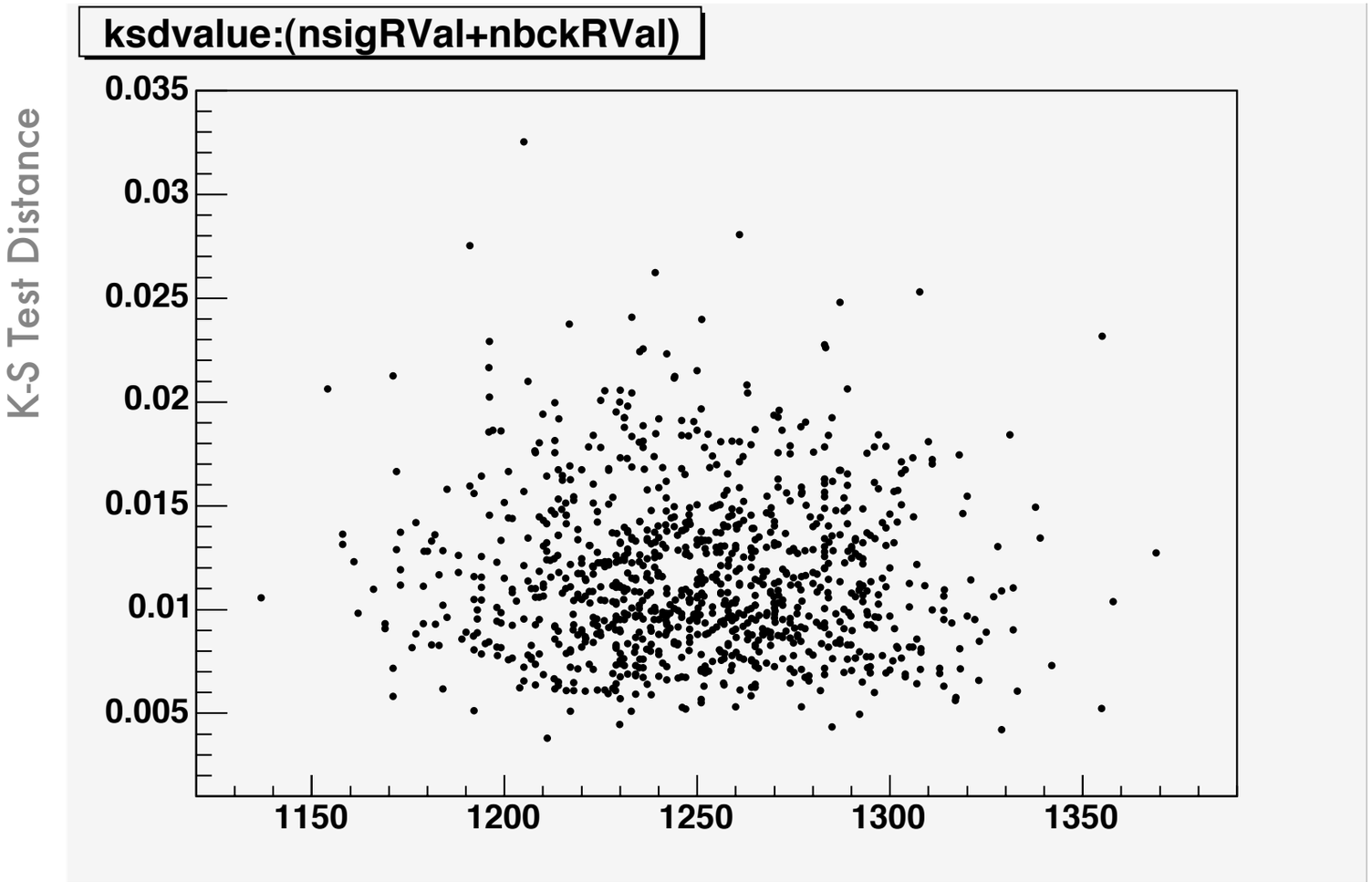
no Goodness of Fit power







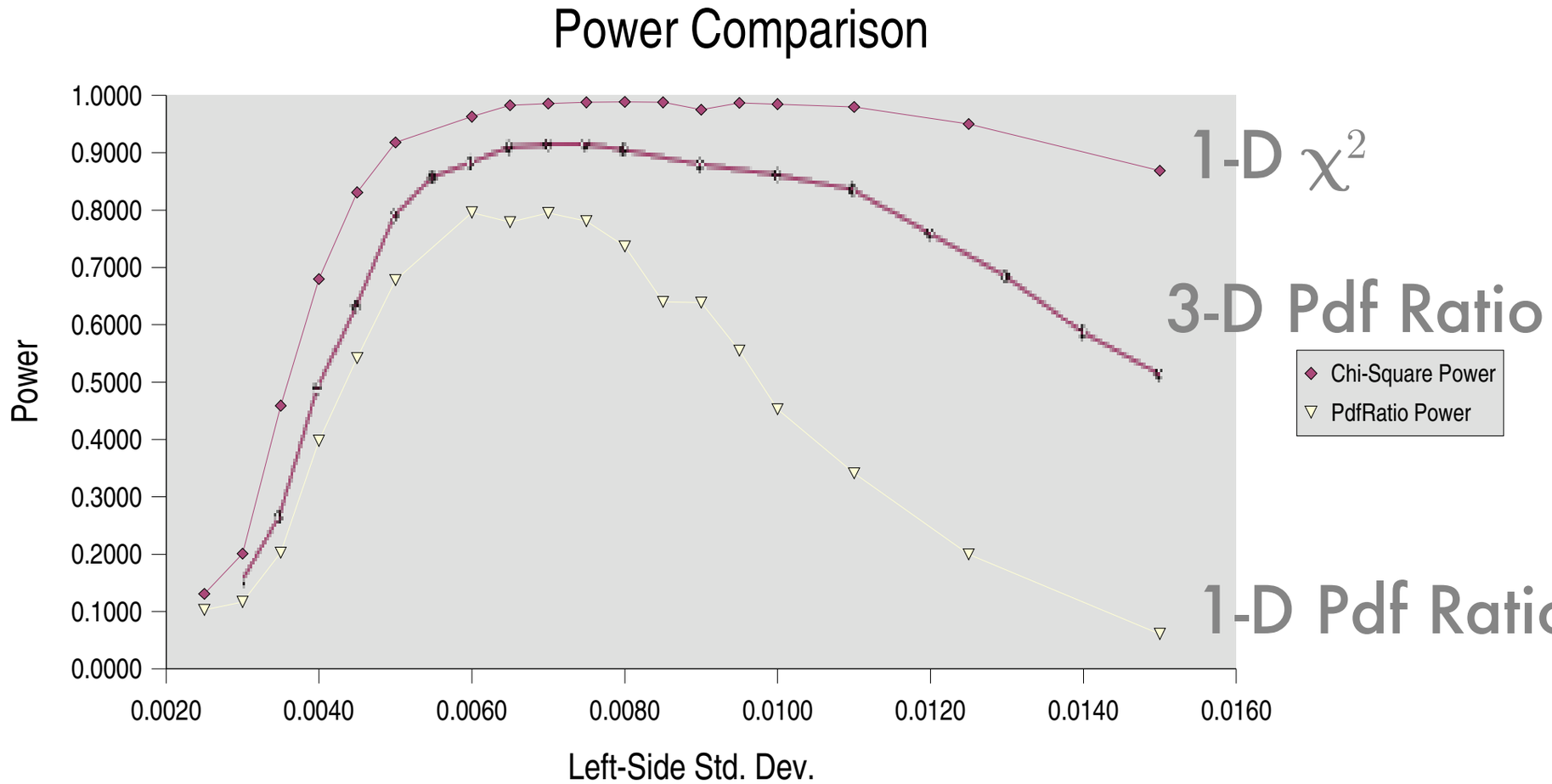
Correlations



no correlation with number of events
(or other fitted parameters)

Goodness of Fit Power?

in 1-D toy study, compare PDF Ratio to χ^2



1-D χ^2

3-D Pdf Ratio

1-D Pdf Ratio

not as good as χ^2 in 1-D case

only sensitive where signal and background overlap

Conclusions

- the value of the minimum log(L) has no Goodness of Fit information
- Pdf Ratio comparison, using K-S distance, between data and fitted Pdf is a valid Goodness Of Fit statistic
- You should make the PDF Ratio plot for your Fit