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Theory and Phenomenology of $B \to \gamma e \nu$, $B \to \gamma \gamma$ and $B \to \gamma e e$

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Very rare processes but accessible to a 10^{36} luminosity machine:

$$BR(B \to \gamma e \nu) \sim 10^{-6}$$

$$BR(B \to \gamma \gamma) \sim 3 \times 10^{-8}$$

$$BR(B \to \gamma e e) \sim 10^{-11} \div 10^{-10}$$

Theoretical status & phenomenology:

- Sensisitvity to SM parameters and to New Physics
- Getting an idea of the uncertainties and of what can be improved
- Phase space cuts required

Theoretical Framework (1/2)

- $m_W \gg m_b$

$$A(B \to X) \sim 1 + (\alpha_s \mathrm{L}) + \alpha_s + (\alpha_s \mathrm{L})^2 + \alpha_s(\alpha_s \mathrm{L}) + \alpha_s^2 + O(\alpha_s^3)$$

$$\mathrm{L} = \log \frac{p_{\mathrm{ext}}^2}{m_W^2}$$

$$A(B \to X) = \langle X | H_{\mathrm{eff}} | B \rangle + O\left(\frac{p_{\mathrm{ext}}^2}{m_W^2}\right) = \sum_i \underbrace{\overbrace{C_i(\mu)}^{\log(\mu^2/m_W^2)} \overbrace{\langle X | O_i(\mu) | B \rangle}^{\log(p_{\mathrm{ext}}^2/\mu^2)}} + O\left(\frac{p_{\mathrm{ext}}^2}{m_W^2}\right)$$

⬇

☆ Using RGE we can resum the log's: $C_i(\mu_b) \langle X | O_i(\mu_b) | B \rangle$

☆ Factorization of Short and Long Distance contributions:
the Wilson coefficients do not depend on the final states

Theoretical Framework (2/2)

- $m_b \gg \sqrt{\Lambda_{QCD}\, m_b} \gg \Lambda_{QCD}$

  - The scales m_b^2 and $\Lambda_{QCD}\, m_b$ are perturbative:

$$\langle X | O(\mu_b) | B \rangle = C(\sqrt{\Lambda_{QCD}\, m_b})\; J(\Lambda_{QCD}) * \langle X | \overline{O}(\Lambda_{QCD}) | B \rangle + O(\frac{\Lambda_{QCD}}{m_b})$$

$$m_b \longrightarrow \sqrt{\Lambda_{QCD}\, m_b} \longrightarrow \Lambda_{QCD}$$

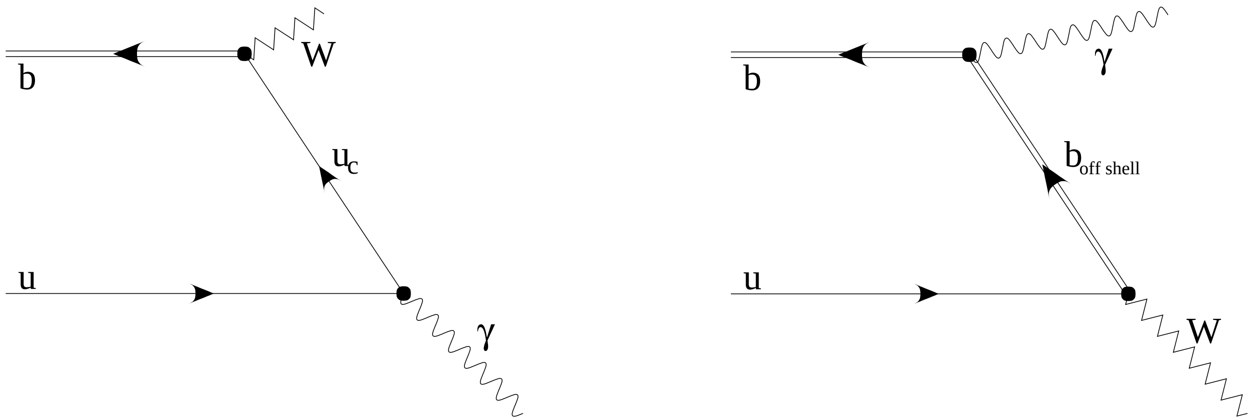
Wilson coefficients	Jet functions	Form Factors Wave functions

- The effective theory apporach (SCET) is as rigorous as in the previous slide
- The Λ_{QCD}/m_b expansion is necessary in order to be able to write the effective theory at all
- For some decays the matrix elements $\langle X | \overline{O}(\Lambda_{QCD}) | B \rangle$ are expressed in terms of light-cone wave functions and/or obey symmetry relations (QCD factorization)

$B \to \gamma e \bar{\nu}$

- The simpler decay $B \to e \bar{\nu}$ is chiral suppressed
- This chiral suppression can be avoided allowing for a final state photon
- The effective Hamiltonian arises at tree level in the SM:

$$H_{\text{eff}} = 4 G_F / \sqrt{2} \; V_{ub} \; (\bar{u}_L \gamma^\mu b_L) \; (\bar{e}_L \gamma_\mu \nu_L)$$



- Sensitive to V_{ub}
- Not expected to receive large New Physics contributions (e.g. from SUSY)
- Gives valuable pieces of information on the B meson wave function

$$A[B^- \to \gamma\, e\, \bar{\nu}] = \frac{4G_F}{\sqrt{2}} V_{ub} \; \langle \gamma |\, \bar{u}\, \gamma_\mu P_L\, b | B^- \rangle \; (\bar{e}\, \gamma^\mu P_L\, \nu)$$

contact term

$$\frac{1}{e} \langle \gamma(q,\varepsilon) | \bar{u} \gamma_\mu b | B(\mathrm{v}) \rangle = i \epsilon_{\mu\alpha\beta\delta} \varepsilon^\alpha \mathrm{v}^\beta q^\delta \; f_V(E_\gamma)$$

$$\frac{1}{e} \langle \gamma(q,\epsilon) | \bar{u} \gamma_\mu \gamma_5 b | B(\mathrm{v}) \rangle = [q_\mu (\mathrm{v}\cdot\varepsilon) - \varepsilon_\mu (\mathrm{v}\cdot q)] \; f_A(E_\gamma) + (\mathrm{v}\cdot\varepsilon) \mathrm{v}_\mu \frac{1}{\mathrm{v}\cdot q} f_B m_B$$

Analysis at all orders in α_s and at leading order in Λ_{QCD}/E_γ

$$\Lambda_{QCD} \ll E_\gamma^c < E_\gamma < \frac{m_B}{2}$$

- Equality of the form factors: $f_V(E_\gamma) = f_A(E_\gamma)$

- Factorization: form factors as a 1-dimensional convolution of perturbative hard scattering kernels and the B meson light cone wave function:

$$f(E_\gamma) = \int d\xi \; T(E_\gamma, \xi) \; \phi_B(\xi) = C(E_\gamma) \int d\xi \; J(E_\gamma, \xi) \; \phi_B(\xi)$$

[Descotes-Genon, Sachrajda: proof at the 1-loop level]
[Lunghi, Pirjol, Wyler; Bosch, Hill, Lange, Neubert: all order proof]

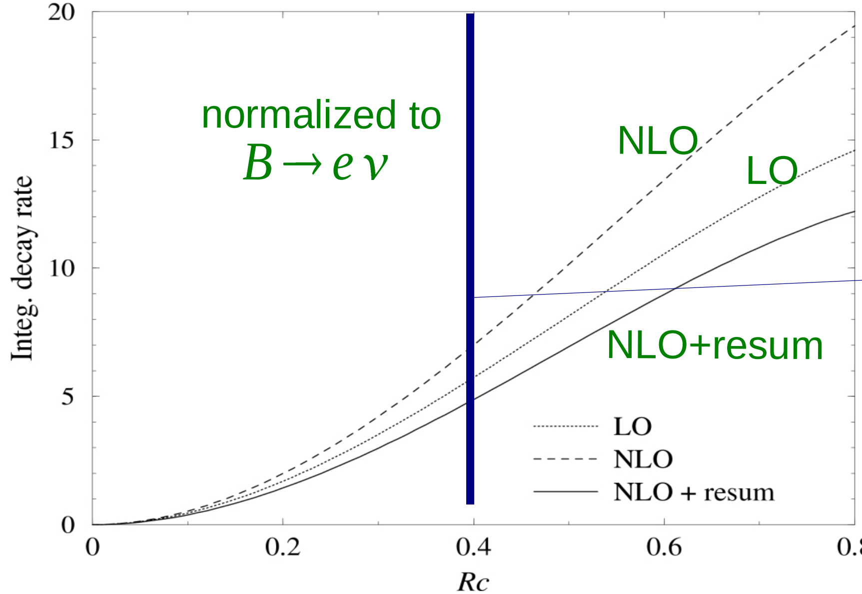
Branching Ratio

- Integrated decay width at LO:

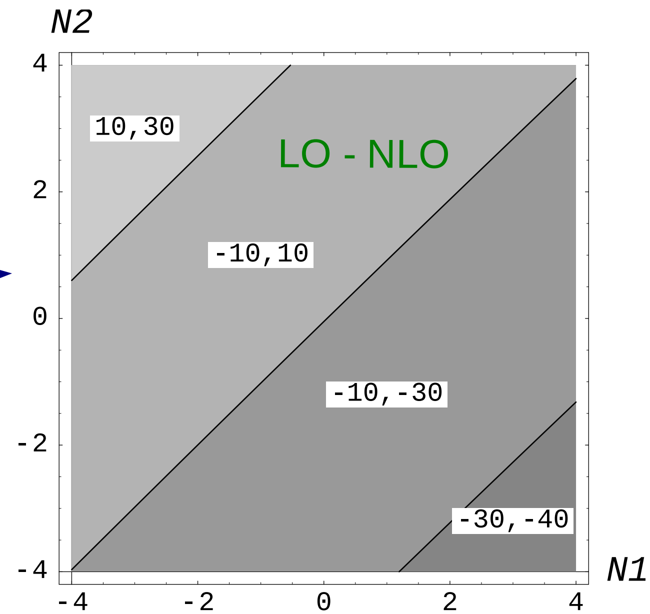
$$BR_{LO} = \underbrace{\tau_{B^+} \frac{\alpha_e G_F^2 |V_{ub}|^2 m_B^5 f_B^2 Q_u^2}{288\pi^2}}_{8.5\times 10^{-7}} \frac{1}{\lambda_b^2} \geq 3\times 10^{-6}$$

$$\frac{1}{\lambda_b} = \int \phi_B(\xi)/\xi \geq \frac{3}{4\bar{\Lambda}} \sim 1.9 \text{ GeV}^{-1}$$

- Sensitivity to the photon energy cut-off and to NLO corrections:



$R_c = 0.4$



$$R_c = 1 - 2E_\gamma^c / m_B$$

N1 and N2 are the first 2 log moments of the B meson light cone wave function

Relevance for QCD

- Use other V_{ub} determinations to extract informations on the B meson wave function

- The tree-level amplitude is proportional to $\lambda_b^{-1} = \int \phi_B(\xi)/\xi$
  The same parameter enter many other B decays:
  $B \to (K^* e \nu\,,\ \rho e \nu\,,\ K^* \gamma\,,\ \rho \gamma\,,\ \pi \pi\,,\ KK\,,\ K \pi\,, \ldots)$

- At 1-loop order the convolution integral involve other logarithmic moments of the B meson wave function: the size of the effect depends on the shape of the wave function itself

- The decays $B \to \gamma \gamma$ and $B \to \gamma e e$ depend on exactly the same convolution integral at all order in perturbation theory

The ratios $\Gamma(B \to \gamma \gamma)/\Gamma(B \to \gamma e \nu)$ and $\Gamma(B \to \gamma e e)/\Gamma(B \to \gamma e \nu)$ are free of hadronic uncertainties up to Λ_{QCD}/E_γ corrections

$B \to \gamma\gamma$ & $B \to \gamma e e$

- FCNC processes: the effective Hamiltonian arises at the loop level in the SM:

$$H_{\text{eff}} = \frac{4G_F}{\sqrt{2}} \left(V_{tb} V_{td}^* \sum_{i=1}^{10} C_i O_i + V_{ub} V_{ud}^* \sum_{i=1}^{2} C_i O_i^u \right)$$

$$O_2 = \bar{d}_L \gamma^\mu c_L \; \bar{c}_L \gamma_\mu b_L \qquad O_9 = \bar{d}_L \gamma^\mu b_L \; \bar{e} \gamma_\mu e$$

$$O_7 = \frac{e}{16\pi^2} m_b \; \bar{d}_L \sigma^{\mu\nu} b_R \; F_{\mu\nu} \qquad O_{10} = \bar{d}_L \gamma^\mu b_L \; \bar{e} \gamma_\mu \gamma_5 e$$

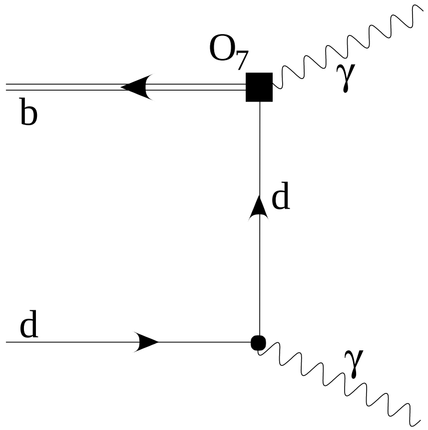
$$O_8 = \frac{g_s}{16\pi^2} m_b \; \bar{d}_L \sigma^{\mu\nu} b_r \; G^a_{\mu\nu}$$

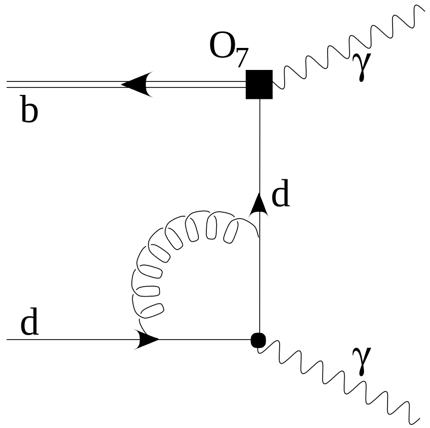
- Same effective Hamiltonian as for $b \to d\gamma$ and $b \to d e e$

- Strong sensitivity to new physics

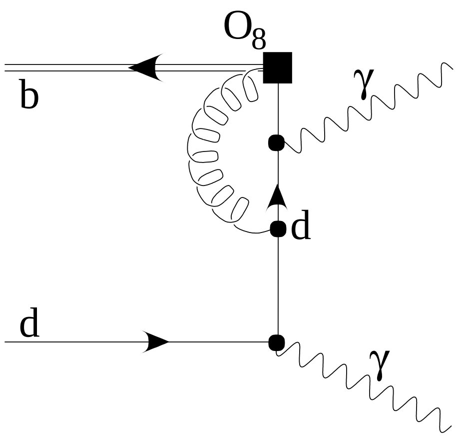
$B \to \gamma\,\gamma$

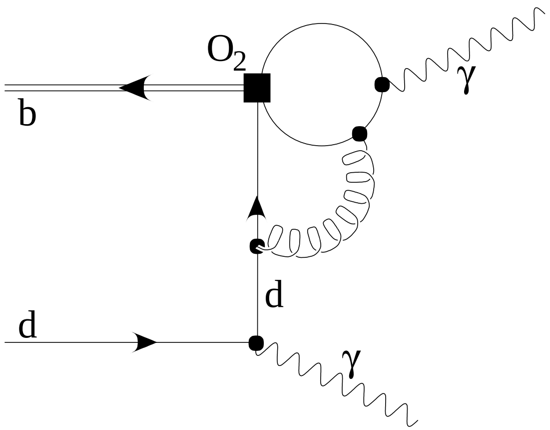
- Contributions from the magnetic moment operator:

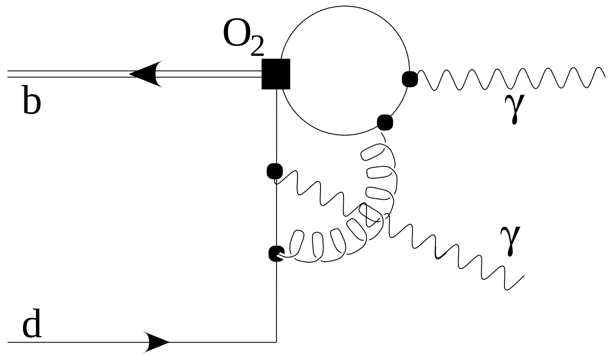




- Contributions from the other operators:







- Traditional approach: only O_7

$$\langle \gamma\gamma | O_7 | B \rangle \propto \langle \gamma | \bar{d}\, \sigma^{\mu\nu} b | B \rangle \rightarrow g_+(E_\gamma),\ g_-(E_\gamma),\ g_0(E_\gamma)$$

- Both photons are energetic and we can apply the effective theory approach

  - the matrix elements of O_2 and O_8 are proportional to the O_7 one up to power corrections:
    [Descotes-Genon, Sachrajda: proof at order α_s]

$$\langle \gamma\gamma | O_{2,8} | B \rangle = (**)\ \langle \gamma\gamma | O_7 | B \rangle + O(\frac{\Lambda_{QCD}}{m_b})$$

m_b^2 fluctuations (perturbative)

  - Factorization of the form factors
    [Lunghi, Pirjol, Wyler; Descotes-Genon, Sachrajda]

$$g_A(E_\gamma) = \int d\xi\ T_A(E_\gamma,\xi)\ \phi_B(\xi) = C_A(E_\gamma) \underbrace{\int d\xi\ J(E_\gamma,\xi)\ \phi_B(\xi)}$$

same convolution as in $b \rightarrow \gamma e \nu$

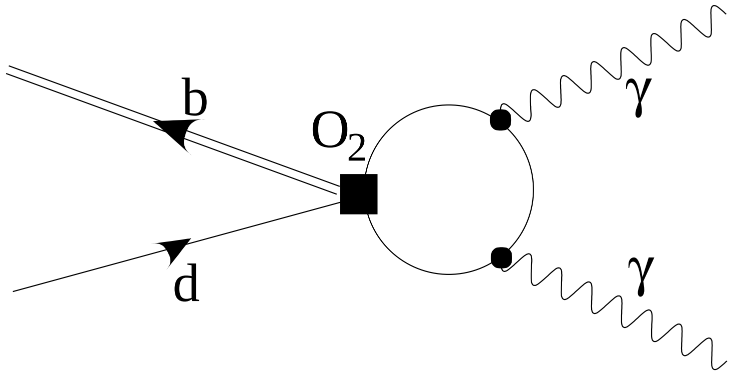
- Explicit form of the symmetry breaking corrections at order α_s

$$\frac{g_+(E_\gamma)}{f_V(E_\gamma)} = \frac{1}{2}\frac{Q_d}{Q_u}\left(1-\frac{\alpha_s C_F}{4\pi}\frac{E_\gamma}{E_\gamma - m_b/2} log \frac{2E_\gamma}{m_b}\right) + o(\alpha_s^2)$$

$$g_-(E_\gamma) = -g_+(E_\gamma) + o(\alpha_s^2)$$

$$g_0(E_\gamma) = 0 + o(\alpha_s^2)$$

- Some power suppressed contributions are computable
  [Bosch, Buchalla: proof at order α_s]



Important for the direct CP asymmetry induced by the $c\bar{c}$ rescattering

- Parameterization of the amplitude

$$A(B\to\gamma\gamma) = \frac{G_F}{\sqrt{2}}\frac{\alpha_e}{\pi} m_B \left[(m_B^2 \epsilon_1\cdot\epsilon_2 - k_1\epsilon_2\cdot k_2\cdot\epsilon_1) A_+ + \varepsilon(k_1,k_2,\epsilon_1,\epsilon_2) A_-\right]$$

$$A_+ = A_- = V_{tb}V_{td}^* \; C_7 \; g_+(m_B/2)$$

A_+: $\propto \vec{\epsilon}_1\cdot\vec{\epsilon}_2$, $CP(\gamma\gamma)=+1$

A_-: $\propto \vec{\epsilon}_1\times\vec{\epsilon}_2$, $CP(\gamma\gamma)=-1$

- Observables

$$\Gamma(B\to\gamma\gamma) = \frac{G_F^2 m_B^5 \alpha_e^2}{8\pi^3}\left(|A_+|^2+|A_-|^2\right)$$

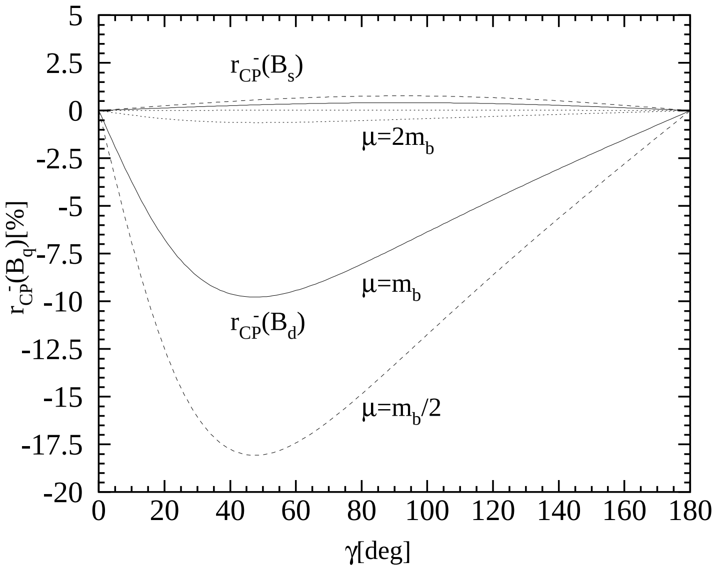
$$r_{CP}^{\pm} = \frac{|A_\pm|^2-|\overline{A}_\pm|^2}{|A_\pm|^2+|\overline{A}_\pm|^2}$$

- A_- receives a power suppressed strong phase:

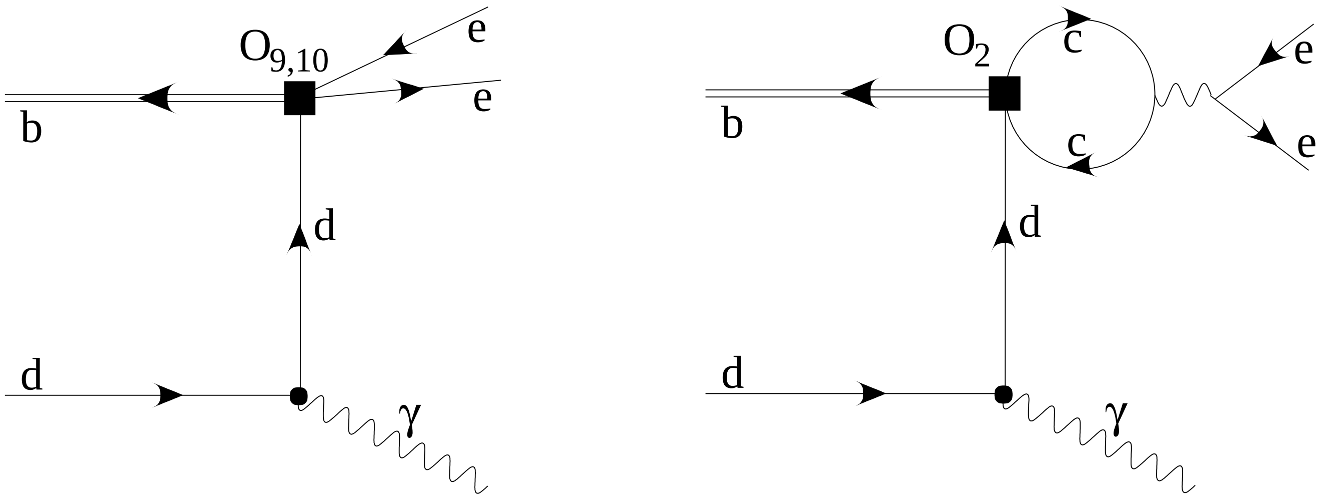
$$r_{CP}^{-} \sim -10\%$$

$$r_{CP}^{+} \sim 0$$

[Bosch, Buchalla]



$B \to \gamma e e$



- New feature: leading order long distance $c\bar{c}$ rescattering $(J/\psi, \psi', ...)$

  Cuts in the dilepton mass spectrum (same as in $b \to s e e$)

- low-s region: the photon energy is large and the SCET approach is feasible

- high-s region: the photon is soft and other methods have to be used

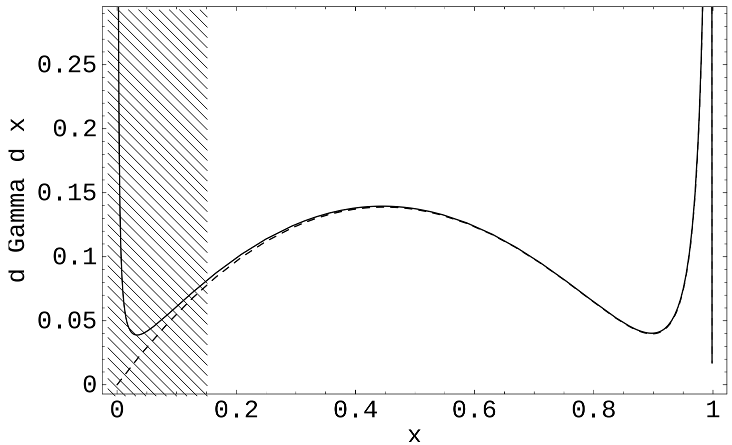
  [heavy quark symmetry, ...]

$$A(B\to\gamma e e) = \frac{G_F}{\sqrt{2}}\frac{\alpha_e}{\pi} V_{tb}V_{td}^* \Big[(C_9^{\rm eff}\,\bar e\gamma^\mu e + C_{10}\,\bar e\gamma^\mu\gamma_5 e)\,\overbrace{\langle\gamma|\bar d_L\gamma_\mu b_L|B\rangle}^{f_V,\,f_A}$$

$$- 2C_7^{\rm eff}\frac{m_b}{q^2}q^\nu\,\underbrace{\langle\gamma|\bar d_L\sigma_{\mu\nu}b_L|B\rangle}_{g_+,\,g_-,\,g_0}\,\bar e\gamma^\mu e \Big]$$

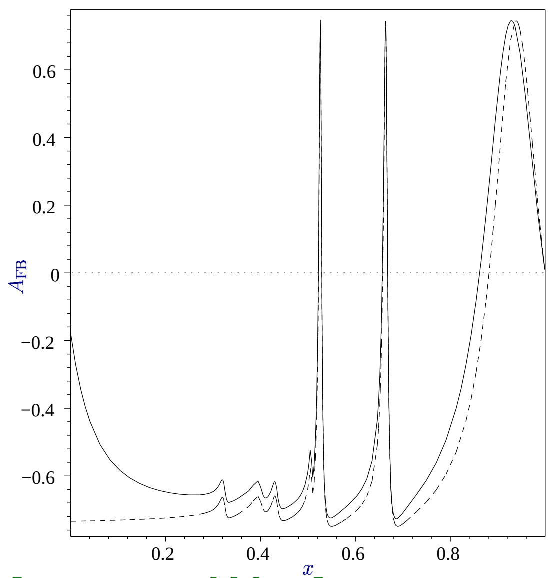
- The ratios $\gamma e e/\gamma e \nu$ and $\gamma e e/\gamma\gamma$ are free of hadronic uncertainties up to power corrections

- Bremmstrahlung from the external leptons



[Dincer, Sehgal] $s/m_B^2 = 1 - x$

- 0 of the forward-backward asymmetry



[Krüger, Mehlikov]

Conclusions

- Our theoretical understanding of these decays is quite good

- Up to power corrections they are all proportional to the same hadronic quantity (i.e. ratios are clean)

- Some power corrections are computable ($B \to \gamma\gamma$)

- $B \to \gamma\gamma$ and $B \to \gamma e e$ are strongly sensitive to new physics in $b \to d$

  Are the coming before or after $B \to X_d \gamma$ and $B \to X_d e e$?

- Clean source of information on the B meson wave function