

# "Selected Topics in Semileptonic B Decays" or "Gearing up for the ~ 1% Challenge"

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We are witnessing B physics adding **high accuracy** to high sensitivity driven by

- magnificent **experim. facilities** **challenging** and thus **inspiring**
- **theoretical technologies**

→ Justification for Super-B factory as **3rd generation** facility  
qualitatively different from B factories as **2nd genen.** facility  
**higher statistics** **fi precision** tool

- ✍ **more** accuracy ... on CKM parameters  $V(ub)$ ,  $V(td)$
- ✍ **more** decays ...  $B \rightarrow e/mn D$ ,  $B \rightarrow tn D$ ,  $B \rightarrow g X_q$
- ✍ **new** territory ...  $^{\circ}(5s) \rightarrow B_s B_s$

no need to apply if **important**, yet **hard measurements** deter you!

can we answer the "1 % challenge"?

predict

measure

interpret

diagnose

observables with  $\sim O(1\%)$  uncert.

assumptions justified on the  $\alpha(10\%)$  accuracy level might **no longer** be adequate on the  $\alpha(1\%)$  level!

Case study on  $V(ub)$  to illustrate the paradigm

# The Menu

- $|V(cb)|, |V(ub)|$  [ $|V(td)|$ ]
- supporting measurements
  - moments, experim. cuts
  - g spectrum
- quality control
  - higher moments
  - $B_d$  vs.  $B_u$  vs.  $B_s$
  - $D_J$  resonan. & Sum Rules
- New Physics &  $B \rightarrow \ell \ell \ell$

numbers & details  
less important here  
-- for they can & will change! --

emphasis on **general strategy** for **high accuracy paradigm**:

- build rich data base involving **hard measurements**
- work to **overconstrain** as much as possible

## On the power of the OPE

✍ a host of observables expressed by a universal cast of Heavy Quark Parameters (=expect. values of local operators):

$$m_Q, m_p^2, m_G^2, r_D^3, r_{LS}^3, \dots$$

memento: only 2 local operators in  $O(1/m_Q^3)$  !

✍ energy & mass moments yield HQP, which in turn can be used universally

✍ caveat: in general not a 1-to-1 correspondence  
moments  $\in$  HQP

➔ need linear combinations of moments

I |V(cb)|

3 methods

- ① the 'golden' way
- ② the 'gold-plated' way
- ③ the 'Cinderella' story

(1.1) The golden way:  $G(B \in \ln X_c)$  'inclusive'

$G(B \in \ln X_c) = F(V(cb), \text{HQP}: m_Q, m_p^2, \dots) \pm 1 - 2\%, \text{th.}$

Benson, Mannel,  
Uraltsev, IB,  
NuPhy. B665,367

limiting factor: perturb.  
correct. to nonpert. contrib.

**Caveat:** do **not** rely on expansion in  $1/m_{ch}$ !  
→ do **not** impose constraint  
 $m_b - m_c = \langle M_B \rangle - \langle M_D \rangle + m_p^2 (1/2 m_{ch} - 1/2 m_b) + \text{nonlocal op.}$   
can check it *a posteriori*

$$\begin{aligned} \rightarrow |V(cb)/0.042| = & 1 - 0.65[m_b - 4.6 \text{ GeV}] - 0.61(m_c - 1.15 \text{ GeV}) \\ & + 0.06(m_G^2 - 0.35 \text{ GeV}^2) - 0.013(m_p^2 - 0.4 \\ & \text{GeV}^2) \\ & - 0.1(r_D^3 - 0.12 \text{ GeV}^3) - 0.01(r_{LS}^3 + 0.15 \text{ GeV}^3) \end{aligned}$$

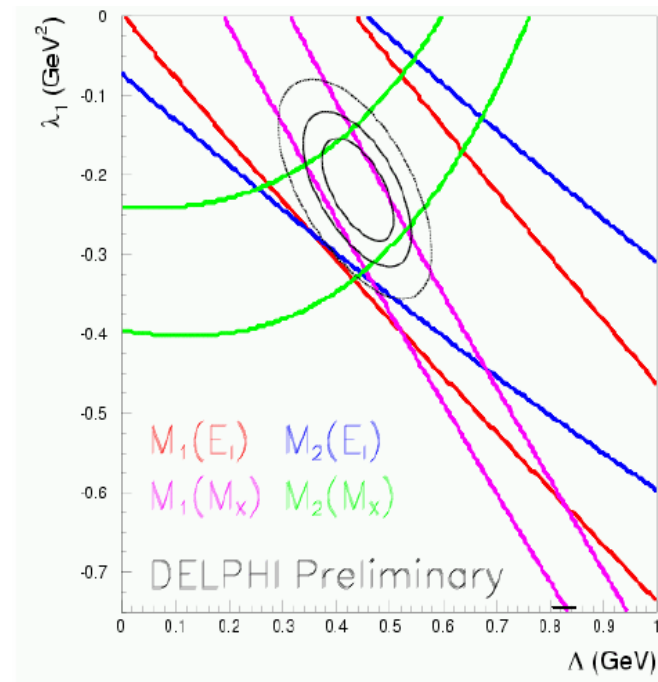
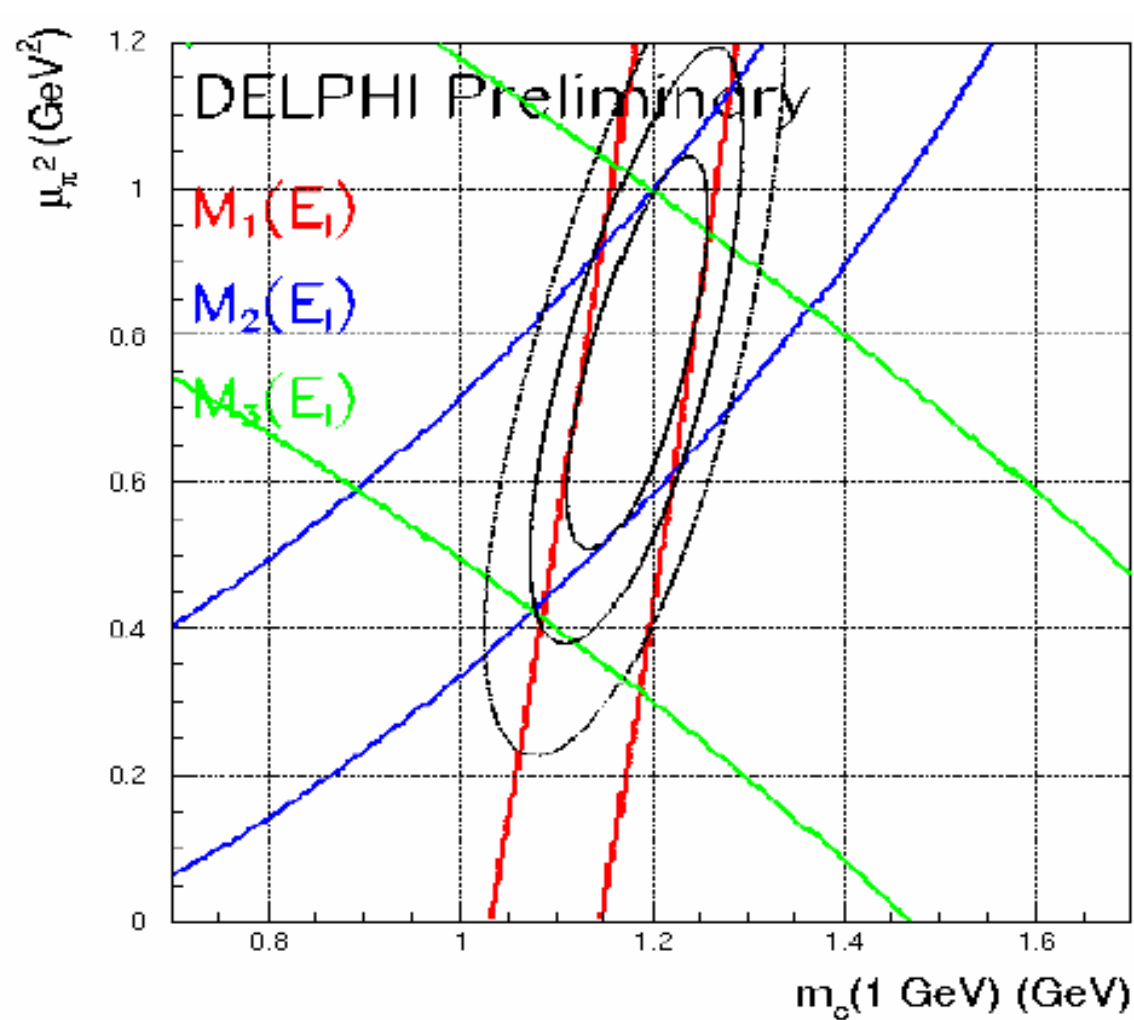
energy/had. mass moments  $\mathcal{A}$  HQP

$$M_1(E_l) = G^{-1} \int dE_l E_l dG/dE_l$$

$$M_n(E_l) = G^{-1} \int dE_l [E_l - M_1(E_l)]^n dG/dE_l, n > 1$$

$$M_1(M_X) = G^{-1} \int dM_X^2 (M_X^2 - M_D^2) dG/dM_X^2$$

$$M_n(M_X) = G^{-1} \int dM_X^2 (M_X^2 - \langle M_X^2 \rangle)^n dG/dM_X^2, n > 1$$



→ need **higher** moments (reason # 1)

$$\rightarrow |V(cb)/0.042| = 1 - 0.65[m_b - 4.6 \text{ GeV}] - 0.61(m_c - 1.15 \text{ GeV}) + 0.06(m_G^2 - 0.35 \text{ GeV}^2) - 0.013(m_p^2 - 0.4 \text{ GeV}^2) - 0.1(r_D^3 - 0.12 \text{ GeV}^3) - 0.01(r_{LS}^3 + 0.15 \text{ GeV}^3)$$

$$|V_{cb}| = 0.0416 \text{ ¥ } (1 \pm 0.017)_{\text{exp}} \pm 0.010_{\Gamma(B)} \pm 0.015_{\text{HQP}}$$

Achille

"us"

vs. "dm<sub>b</sub> ~ 2% implying δ|V<sub>cb</sub>| > 5%"???

low moments depend on ~ same comb. of HQP!

$$[.=F(m_b - 0.65m_c)]$$

$$[\Gamma_{\text{part}} \mu m_b^2 (m_b - m_c)^3]$$

$$|V(cb)/0.042| = 1 - 1.71 [\langle E_l \rangle - 1.38 \text{ GeV}] - 0.06 (m_c - 1.15 \text{ GeV}) + 0.08 (m_G^2 - 0.35 \text{ GeV}^2) - 0.07 (m_p^2 - 0.4 \text{ GeV}^2) - 0.05 (r_D^3 - 0.12 \text{ GeV}^3) - 0.005 (r_{LS}^3 + 0.15 \text{ GeV}^3)$$

"caveat emptor" : relation  $\{\text{moments} \leftrightarrow \text{HQP}\}$

has not been scrutinized **yet** to same degree as

relation  $\{G_{SL}(B) \leftrightarrow \text{HQP}\}$  (s.later)

(1.2) The gold-plated way:  $B \rightarrow \ln D^*$  at zero recoil - 'exclusive'

measure rate of  $B \rightarrow \ln D^*$

- ☺ extrapolate to zero recoil & extract  $|V(cb) F_{D^*}(0)|$
- ☺  $F_{D^*}(0) = 1 + \mathcal{O}(1/m_Q^2) + \mathcal{O}(a_s)$  normalized
- ☺ holds automatically for  $m_b = m_c$
- ☹ expansion in  $1/m_c!$

$$F_{D^*}(0) = \begin{cases} 0.89 \pm 0.08 [0.05] & \text{Uraltsev et al.: } \mathcal{O}(1/m_Q^2) \\ 0.913 \pm 0.042 & \text{BaBar Book: not a consensus!} \\ 0.935 \pm 0.03 & \text{first } \textit{prelim.} \text{ lattice: Hashimoto, Kronf. et al.} \\ 0.913^{+0.024}_{-0.017} {}^{+0.017}_{-0.030} & \text{2nd } \textit{quenched} \text{ lattice: H,K et al. } \mathcal{O}(1/m_Q^3) \\ & [\sim 0.89 \text{ at } \mathcal{O}(1/m_Q^2)] \end{cases}$$

**caveat:** relies on expansion in  $1/m_c!$

will use:  $F_{D^*}(0) = 0.90 \pm 0.05$  for convenience

$$|F_{D^*}(0)V(\text{cb})| = 0.0367 \pm 0.0013$$

$$\Rightarrow |V(\text{cb})|_{\text{excl}} = 0.0408 \text{ } \forall (1 \pm 0.035|_{\text{exp}} \pm 0.06|_{\text{theor}})$$

vs.

$$|V(\text{cb})|_{\text{incl}} = 0.0416 \text{ } \forall (1 \pm 0.017|_{\text{exp}} \pm 0.010|_{\Gamma(B)} \pm 0.015|_{\text{HQP}})$$

😊 full agreement with different systematics!

☹ theor. "brickwall" for  $|V(\text{cb})|_{\text{excl}}$ : expansion in  $1/m_c!$

(1.3) A 'Cinderella' story in the making?  $B \in D$  - 'exclusive'

II

$B \in D$  seen as 'poor relative' of  $B \in D^*$

- $F_{B \in D}(0)$  has  $1/m_c$  term -- unlike for  $F_{B \in D^*}(0)$
- $F_{B \in D}(0)$  -- unlike  $F_{B \in D^*}(0)$  -- **not** normal. to unity for  $m_Q \in \bullet$   
another 'cinderella' story in the making?

BPS limit as 'good fairy'

if  $\mu_\pi^2 = \mu_G^2$ :  $\sigma \bullet \pi |B\rangle = 0$ ,  $\rho^2 = 3/4$

→ FF  $f_-(q^2) = -(M_B - M_D)/(M_B + M_D) f_+(q^2)$  modif. by pert. corr.,  
**un**affect. by **power** corrections

in real QCD  $\mu_\pi^2 - \mu_G^2 \ll \mu_\pi^2$ ,  $\rho^2 \approx 3/4$

expansion in  $\beta = [3(\rho^2 - 3/4)]^{1/2} = 3 [\sum_n |\tau_{1/2}^{(n)}|^2]^{1/2}$

**irreducible**  $df_+(0) \sim \exp(-2m_c/\mu_{\text{had}}) \sim \text{few } \%$

## Program:

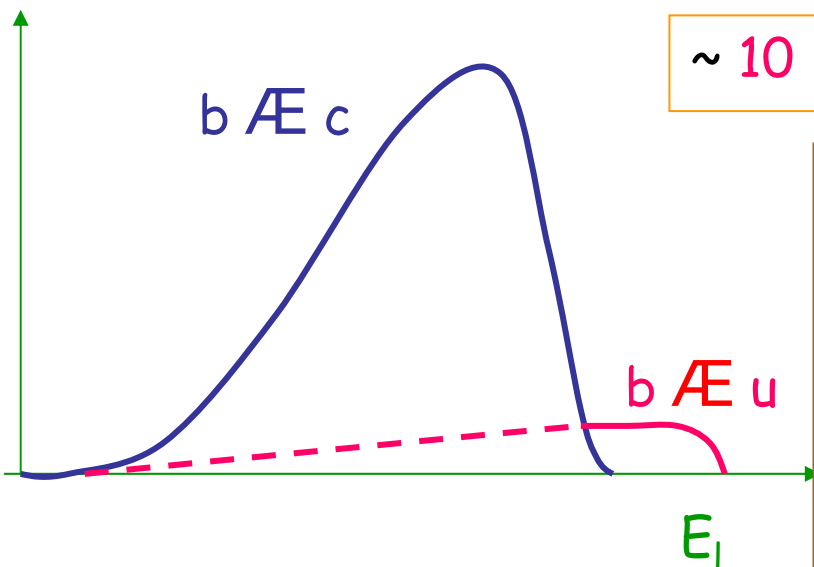
- ① extract  $|V(cb)|$  from  $B \rightarrow e/m n D$
- ② compare with 'true'  $|V(cb)|$  from  $G_{SL}(B)$
- ③ if successful, use it for  $B \rightarrow t n D$  (2nd FF!)  
[ hadronic FF does **not** drop out from  $G(B \rightarrow t n D) / G(B \rightarrow m n D)$  !]
- ④ check for **New Physics** contrib. to  $B \rightarrow t n D$  (Higgs-X)

## II $|V(\text{ub})|$

### Inclusive method

- ☺ expansion in  $1/m_b$  better
- ☺ purely perturb. contributions better known
- ☹ need  $m_b$  rather than  $m_b - 0.65 m_c$  → need higher moments (reason # 2)
- ☹ no HQS in final state, no SV Sum Rules
- ☹ experimentally much harder

# (2.1) Lepton Energy Endpoint Spectrum



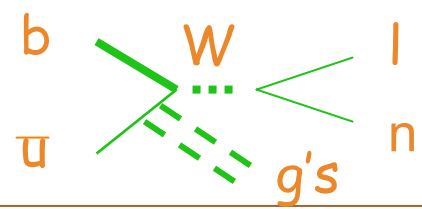
~ 10 % of signal accessible!

✍ model dependence can be reduced  
 inferring shape function from  $B \to g X$  spectrum but far from eliminated:  
 □ subleading correct. to shape fctn  
 + shape fct( $b \to g s$ ) = shape fct( $b \to l n u$ )  
 +  $\mathcal{O}(1/m_b)$

✍ expect sizeable violations to local duality

□ endpoint different for  $B_d$  and  $B^+$  due to WA! Uralt., IB '94

➔ will not be a precision extraction of  $|V(ub)|!$

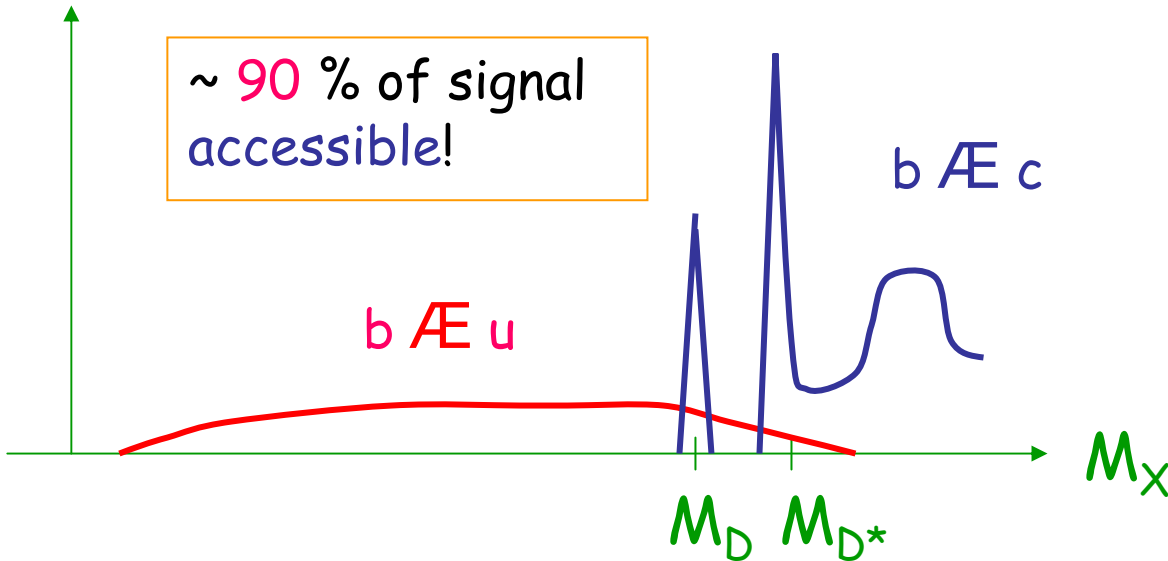


## (2.2) Hadronic Recoil Mass Spectrum

partially integrat. had. recoil mass spectrum

$$\int_0^{M_{X,\max}} dM_X ds/dM_X (B \rightarrow c \bar{c} X) \text{ with } M_{X,\max} < M_D$$

~ 90 % of signal accessible!



mild dependence on cut-off  $M_{X,\max}$  for  $M_{X,\max} \sim 1.6 \text{ GeV}$

least reliable part **theoretically**: low  $q^2$  ( $q$  = lepton pair mom.)

 cut low  $q^2$  **Bauer et al.**

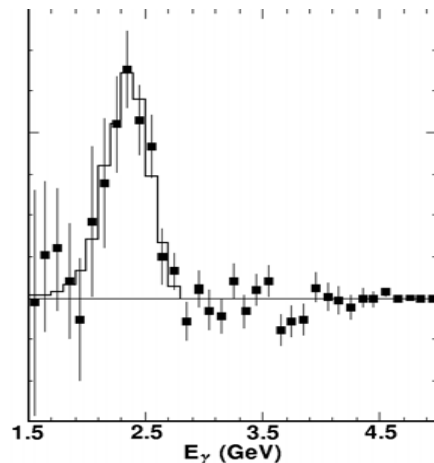
 can be done

 lose constraints due to **Sum Rules**

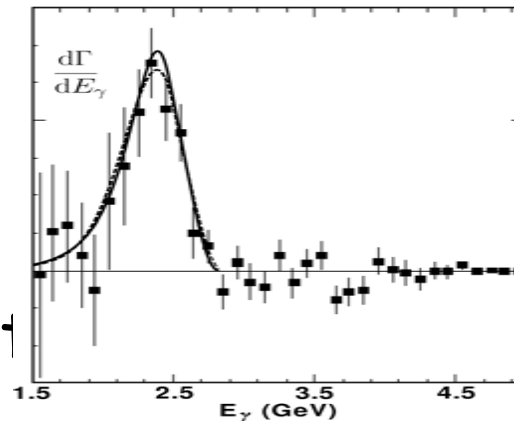
 retain < 50 % of rate -- duality viol.?

 infer from recoil mass spectrum in  **$B \rightarrow e^+ e^- g X$**  **Uraltsev,**  
**IB**

 need **photon spectrum below 2 GeV** - say  $1.8 < E_g < 2 \text{ GeV}$



$\sim 5\%$  at



# III $|V(td)|$

usual avenues:

- $K^+ \text{ } \text{AE} \text{ } p^+ \text{ } n \text{ } n$  best -- theoretically
- $B \text{ } \text{AE} \text{ } g \text{ } r/w$  vs.  $B \text{ } \text{AE} \text{ } g \text{ } K^*$  control over long distance dynamics?
- $B_s - B_s$  vs.  $B_d - B_d$  oscillations

newcomer (?): differentiate between

$$B \text{ } \text{AE} \text{ } g \text{ } X_d \text{ vs. } B \text{ } \text{AE} \text{ } g \text{ } X_s$$

to extract  $|V(td)/V(ts)|$

- ☹ experimentally very challenging
- ☺ theoretically clean --
- ☹ within the SM!

## IV Impact of Experimental Cuts

Experimental cuts on energy etc. applied for practical reasons

yet they degrade 'hardness'  $Q$  of transition

$\exists$  'exponential' contributions  $\exp[-cQ/m_{\text{had}}]$  missed in usual OPE expressions

 quite irrelevant for  $Q \gg m_{\text{had}}$

 yet relevant for  $Q \sim m_{\text{had}}$ !

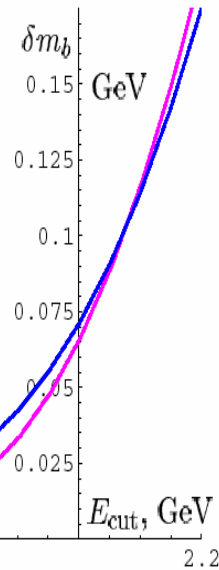
for  $B \rightarrow g X_q$ :  $Q = m_b - 2 E_{\text{cut}}$   
e.g.: for  $E_{\text{cut}} \sim 2 \text{ GeV}$ ,  $Q \sim 1 \text{ GeV}$ !

Pilot study (Uraltsev, IB)

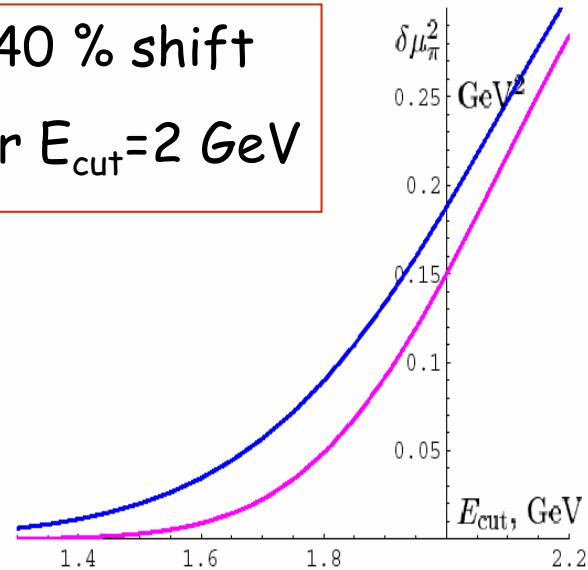
earlier work by C. Bauer

- considers very different effects
- addresses theor. **uncertaint.**, **not biases!**

~ 1.5 % shift  
for  $E_{\text{cut}}=2 \text{ GeV}$



~ 40 % shift  
for  $E_{\text{cut}}=2 \text{ GeV}$

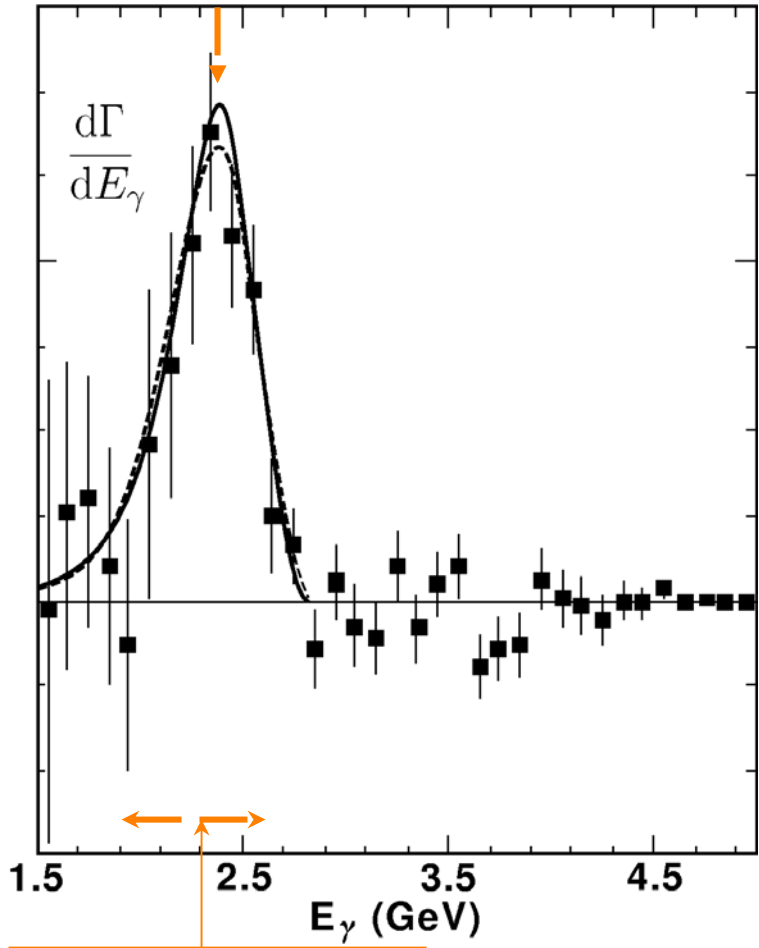
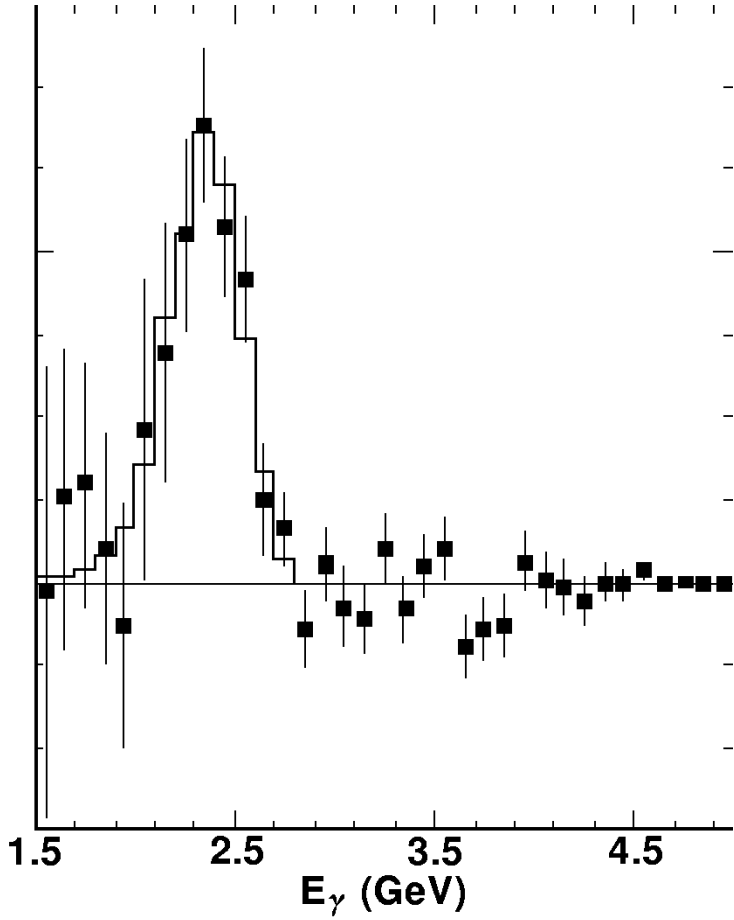


absolute **bias** due to experim. cut

       2 different ansaetze for distribution function

[curves shown for  $m_b=4.6 \text{ GeV}$ ,  $\mu_\pi^2 = 0.45 \text{ GeV}^2$ ; bias depends on HQP]

average  $\bar{E} \approx m_b/2$



width  $\Delta E \approx m_p^2$

terms  $\sim O(1/m_Q^3)$  irrelevant for this analysis

→ only 3 dimensional parameters:  $m_b, \mu_\pi^2, Q = m_b - 2 E_{\text{cut}}$

→ simple scaling behaviour arises

I have created six tables for  $\frac{\delta m_b}{\sqrt{\mu_\pi^2}}$  versus  $\frac{Q}{\sqrt{\mu_\pi^2}}$ . There are three tables for each different ansatz. The three tables are for  $\frac{\mu_\pi^2}{\Lambda^2} = 0.7, 1.0, 1.3$ . I have plotted the tables for you to see.

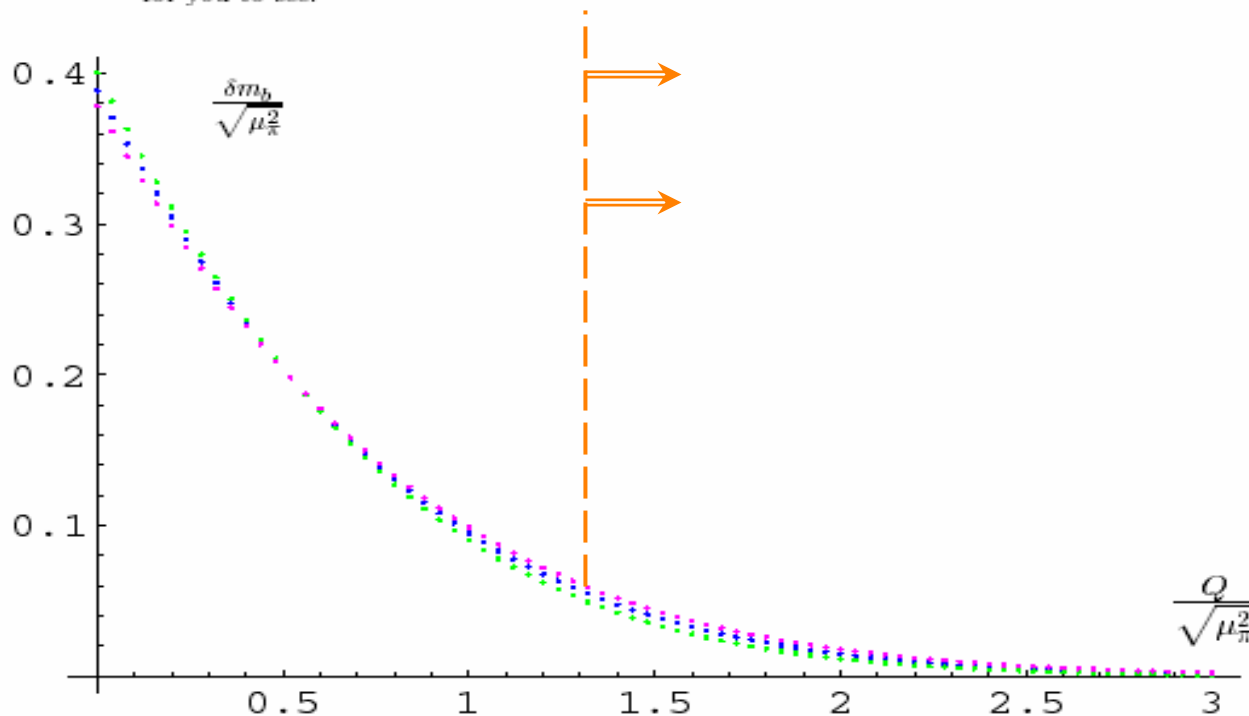
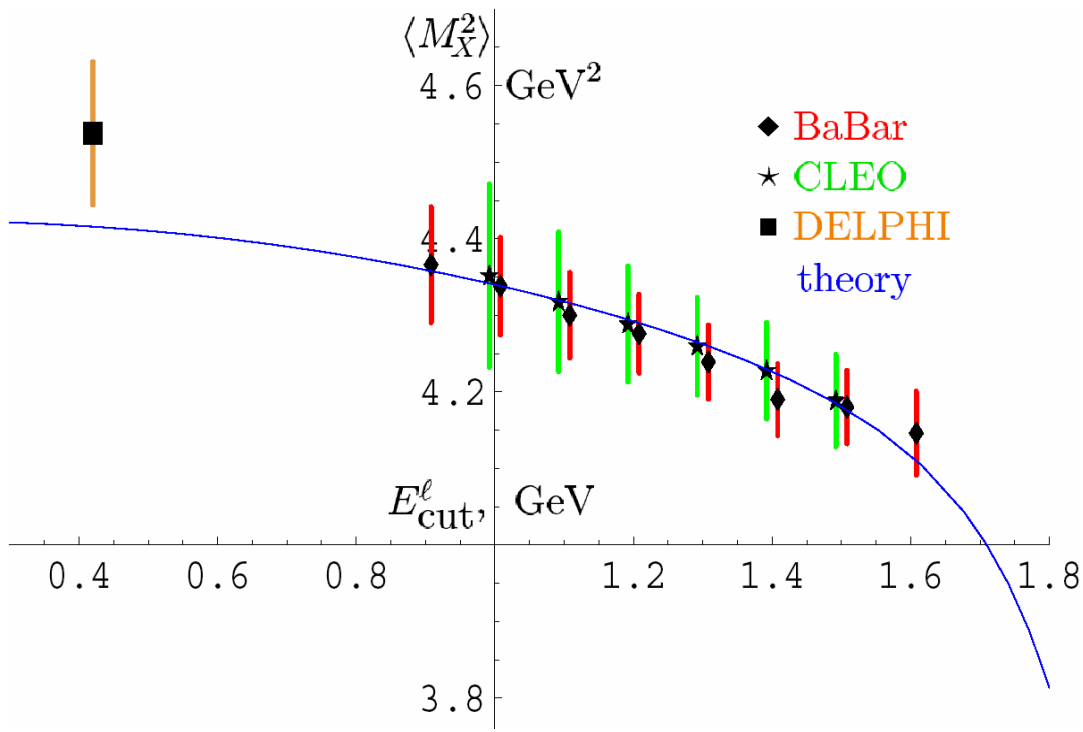


Figure 1:  $\frac{\delta m_b}{\sqrt{\mu_\pi^2}}$  versus  $\frac{Q}{\sqrt{\mu_\pi^2}}$  using the  $e^{-\alpha}$  distribution function. The blue line gives the bias for  $\frac{\mu_\pi^2}{\Lambda^2} = 1$ . The purple and green lines show the variation in the ratio as  $\frac{\mu_\pi^2}{\Lambda^2}$  is varied over  $\pm 30\%$ .

QuickTime™ and a  
TIFF (Uncompressed) decompressor  
are needed to see this picture.



hadronic mass moments in  $B \rightarrow \ell n$   
 $X_c$

➔ encouraging, yet more work needed

**Lessons:**

- ❑ keep the cuts as low as possible
- ❑ bias in the meas. moments induced by cuts
  - ➡ can be corrected for
  - ➡ not a pretext for inflating theor. uncert.
- ❑ moments meas. as fctn of cuts: important cross check!

# V Quality Control

Overconstraints most powerful protection against ignorance  
Lenin's dictum: "Trust is good -- control is better!"

- measure higher (2nd and 3rd) moments
- of different types
- keep energy cuts as low as possible
- yet analyze moments as function of (reasonable) cuts
- final states in  $B \in |v\rangle D(s_q = 1/2 \text{ or } 3/2)$

prod. of diff.  $D(s_q)$

HQP

SSV sum rules



→  $r^2(m) - 1/4 = S_n |t_{1/2}^{(n)}|^2 + 2 S_m |t_{3/2}^{(m)}|^2$

→  $1/2 = -2 S_n |t_{1/2}^{(n)}|^2 + S_m |t_{3/2}^{(m)}|^2$

→  $\bar{L}(m) = 2 ( S_n e_n |t_{1/2}^{(n)}|^2 + 2 S_m e_m |t_{3/2}^{(m)}|^2 )$

→  $m_p^2(m)/3 = S_n e_n^2 |t_{1/2}^{(n)}|^2 + 2 S_m e_m^2 |t_{3/2}^{(m)}|^2$

→  $m_G^2(m)/3 = -2 S_n e_n^2 |t_{1/2}^{(n)}|^2 + 2 S_m e_m^2 |t_{3/2}^{(m)}|^2$

→ ..... where:  $t_{1/2}$  &  $t_{3/2}$  denote transition amplitudes for  
 $B \rightarrow D(s_q = 1/2 \text{ or } 3/2)$   
with excitation energy  $e_k = M(B) - M(D_k)$ ,  $e_k \ll m$

→ rigorous inequalities + experim. constraints

SV SR strongly suggested broad  $D(s_q=3/2)$  to lie below 2400 MeV -- contrary to usual interpretation of data.  
BaBar's discovery of  $D_{sJ}$  reopened issue!

- extract CKM parameters *separately* for  
 $SL B_u$  vs.  $B_d$  vs.  $B_s$  decays  
 as a *check* on quark-hadron duality

*rationale*: nearby & narrow resonance could modify rate

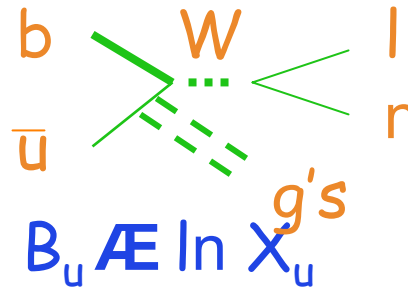
  $b \rightarrow c$ :  $DI = 0$

→ resonance could affect  $B_d$  &  $B_u \rightarrow c$  -- but *equally*

$B_u \rightarrow c$  vs.  $B_d \rightarrow c$ ,  $G_{SL}(B_s)$  would provide test!  
 not doable at had.coll.(?)

  $b \bar{u}$ :  $DI = 1/2$

  $ds(B_d \bar{u} \ln X_u)/dE_l \propto ds(B_u \bar{u} \ln X_u)/dE_l$  in  $\mathcal{O}(1/m_b^3)$ !






  $B_d \bar{u} \ln X_u$  vs.

$B_u \bar{u} \ln X_u$

$I_f = [1, +1]$  vs  $I_f = [1, 0] + [0, 0]$   
 affected differently by a resonance

# VI Summary

1 - 5 % accuracy level in

  $|V(cb)|$  achieved  
  $|V(ub)|$  } not utopian  
  $|V(td)|$  }

requirements

- theoretical analysis challenged and fed by
- rich data base
- that requires  $> 0.5 \text{ ab}^{-1}$  data from B factory
- with constant re-evaluation of assumptions & approxim.

e.g.:

to extract a value for  $|V(ub)|$  with an error estimate that can be defended need

- ✍ measurement of  $\int dM_X ds/dM_X (B \rightarrow \ell n X)$  with  $M_{X,\max} < M_D$
- ✍ preferably for  $B_d$  &  $B_u$  separately and the
- ✍ photon spectrum in  $B \rightarrow \ell g X$  with  $1.8 < E_g < 2 \text{ GeV}$







