
β penguins

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The problem with $b \rightarrow sq\bar{q}$ decays

$$A = \underbrace{V_{cb}V_{cs}^*}_{[\lambda^2]} P + \underbrace{V_{ub}V_{us}^*}_{[\lambda^4]} T$$

dominant contribution

suppressed by λ^2

- We often use $P = |P|e^{i\delta}$ $T = |T|e^{i\gamma}$
- When $\lambda^2 T \ll P$ then A is dominated by a single phase
 - $C_{s\bar{q}q} \approx 0$
 - $S_{s\bar{q}q} \approx S_{\psi K}$
- With new physics: $S_{s\bar{q}q} \neq S_{\psi K}$ and $C_{s\bar{q}q} \neq 0$ possible

How large are the subleading effects in the SM?

Definitions

- Consider $b \rightarrow s\bar{q}q$ decay

$$A_f \equiv A(B^0 \rightarrow f) = V_{cb}^* V_{cs} a_f^c + V_{ub}^* V_{us} a_f^u = V_{cb}^* V_{cs} a_f^c (1 + \xi_f)$$

$$\xi_f \equiv \frac{V_{ub}^* V_{us}}{V_{cb}^* V_{cs}} \frac{a_f^u}{a_f^c}, \quad \left| \frac{V_{ub}^* V_{us}}{V_{cb}^* V_{cs}} \right| = \mathcal{O}(\lambda^2), \quad \delta_f = \arg \frac{a_f^u}{a_f^c}$$

- $S_f - \sin 2\beta \approx 2 \cos 2\beta \sin \gamma \cos \delta_f |\xi_f|$
- $C_f \approx -2 \sin \gamma \sin \delta_f |\xi_f|$

We like to get the value of $|\xi_f|$

Getting ξ_f

We concentrate on $b \rightarrow s$ modes where we expect $|\xi_f| \ll 1$

$$B \rightarrow \phi K_S \quad B \rightarrow \eta' K_S \quad B \rightarrow \pi^0 K_S \quad \dots$$

Two ways

- Calculate (with experimental input)
- Using flavor symmetries

Trying to get as much information as possible. The two ways are both important

An example: $B \rightarrow \eta' K_S$

Beneke and Neubert

$|\xi_{\eta' K_S}|$ was calculated using QCD factorization (BBNS)

$$|\xi_{\eta' K_S}| \approx 0.06 - 0.09$$

- The spread is due to model dependence
- The strong phase was also calculated and found to be small

SU(3) relations: example

- Consider the U-spin pair

$$B \rightarrow \pi^0 K \quad (b \rightarrow sq\bar{q}) \qquad B_s \rightarrow \pi^0 \bar{K} \quad (b \rightarrow dq\bar{q})$$

- Using U-spin $(\lambda \sim 0.22)$

$$A(B \rightarrow \pi^0 K) \equiv P + \lambda^2 T \qquad A(B_s \rightarrow \pi^0 \bar{K}) = \lambda(P + T)$$

- For simplicity we assume

$$A(B \rightarrow \pi^0 K) \approx P \qquad A(B_s \rightarrow \pi^0 \bar{K}) \approx \lambda T$$

- We get

$$\xi = \frac{\lambda^2 T}{P} \approx \lambda \frac{A(B_s \rightarrow \pi^0 \bar{K})}{A(B \rightarrow \pi^0 K)} \approx \lambda \sqrt{\frac{\Gamma(B_s \rightarrow \pi^0 \bar{K})}{\Gamma(B \rightarrow \pi^0 K)}}$$

SU(3) relations

- For $b \rightarrow q\bar{q}s$ transitions

$$A_f = V_{cb}^* V_{cs} a_f^c + V_{ub}^* V_{us} a_f^u = V_{cb}^* V_{cs} a_f^c (1 + \xi_f)$$

- For $b \rightarrow q\bar{q}d$ transitions

$$A_{f'} = V_{cb}^* V_{cd} b_{f'}^c + V_{ub}^* V_{ud} b_{f'}^u = V_{ub}^* V_{ud} b_{f'}^u (1 + \lambda^2 \xi_{f'}^{-1})$$

SU(3) gives relations among a_f^q and $b_{f'}^q$,

$$a_f^u = \sum_{f'} x_{f'} b_{f'}^u$$

$x_{f'}$ are CG coefficients.

SU(3) relations

- The branching ratios $\mathcal{B}(f)$ constrain a_f^c and $b_{f'}^u$

$$|\xi_f| \equiv \left| \frac{V_{ub}^* V_{ud}}{V_{cb}^* V_{cs}} \frac{b_{f'}^u}{a_f^c} \right| \sim \lambda \sqrt{\frac{\mathcal{B}(f')}{\mathcal{B}(f)}}$$

This can be use to get our “best estimate” for $|\xi_f|$

- Combining $SU(3)$ and experimental data gives

$$\hat{\xi}_f \equiv \left| \frac{\xi_f + \lambda^2}{1 - \xi_f} \right| \leq \lambda \sum_{f'} |x_{f'}| \sqrt{\frac{\mathcal{B}(f')}{\mathcal{B}(f)}}$$

This is used to get exact bounds (in the SU(3) limit)

Results: $B \rightarrow \pi^0 K_S$

- SU(3) relation

$$a(\pi^0 K^0) = b(\pi^0 \pi^0) + b(K^+ K^-)/\sqrt{2}$$

- Data: $\mathcal{B}(B^0 \rightarrow \pi^0 K^0) = (11.92 \pm 1.44) \times 10^{-6}$

$$\mathcal{B}(B^0 \rightarrow \pi^0 \pi^0) = (1.89 \pm 0.46) \times 10^{-6}$$

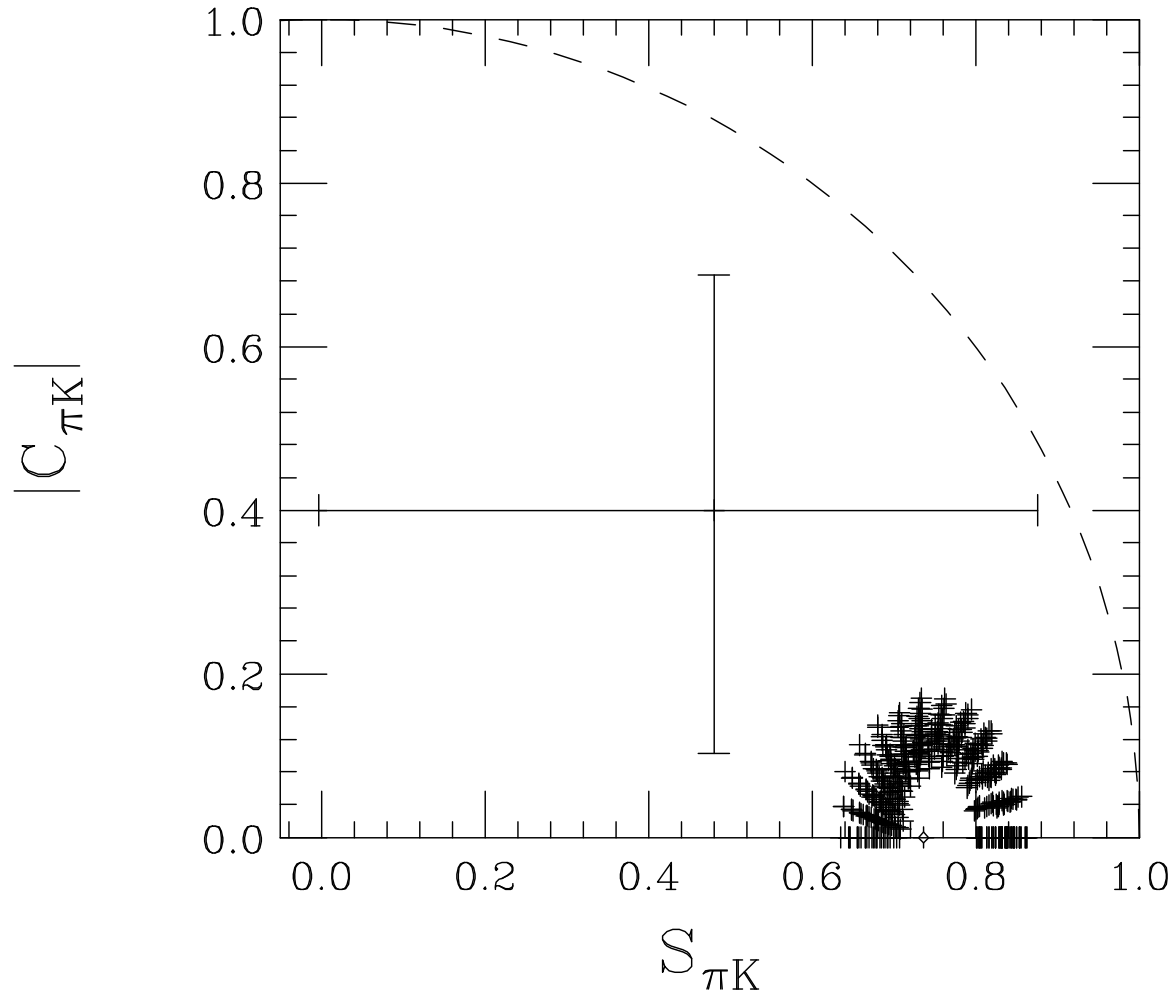
$$\mathcal{B}(B^0 \rightarrow K^+ K^-) < 0.6 \times 10^{-6}$$

- We get

$$\xi \sim 0.08, \quad \hat{\xi} < 0.13, \quad |S_{\pi K} - \sin 2\beta| < 0.19, \quad |C_{\pi K}| < 0.26$$

- We expect $\mathcal{B}(B^0 \rightarrow K^+ K^-)$ to be very small. Neglecting it we get stronger bounds

$B \rightarrow \pi^0 K_S$



- Neglecting $B^0 \rightarrow K^+ K^-$

Results: $B \rightarrow \eta' K_S$

More complicated SU(3) relation

$$\begin{aligned} a(\eta' K^0) &= \frac{s^2 - 2c^2}{2\sqrt{2}} b(\eta' \pi^0) - \frac{3sc}{2\sqrt{2}} b(\eta \pi^0) \\ &+ \frac{\sqrt{3}s}{4} b(\pi^0 \pi^0) - \frac{\sqrt{3}s(s^2 + 4c^2)}{4} b(\eta' \eta') \\ &+ \frac{3\sqrt{3}sc^2}{4} b(\eta \eta) + \frac{\sqrt{3}c(2c^2 - s^2)}{2\sqrt{2}} b(\eta \eta') \end{aligned}$$

$$s \equiv \sin \theta_{\eta\eta'}, \quad c \equiv \cos \theta_{\eta\eta'}$$

Results: $B \rightarrow \eta' K_S$

Which gives

$$\begin{aligned} \hat{\xi}_{\eta' K_S} < \lambda \left[0.59 \sqrt{\frac{\mathcal{B}(\eta' \pi^0)}{\mathcal{B}(\eta' K^0)}} + 0.33 \sqrt{\frac{\mathcal{B}(\eta \pi^0)}{\mathcal{B}(\eta' K^0)}} \right. \\ \left. + 0.14 \sqrt{\frac{\mathcal{B}(\pi^0 \pi^0)}{\mathcal{B}(\eta' K^0)}} + 0.53 \sqrt{\frac{\mathcal{B}(\eta' \eta')}{\mathcal{B}(\eta' K^0)}} \right. \\ \left. + 0.38 \sqrt{\frac{\mathcal{B}(\eta \eta)}{\mathcal{B}(\eta' K^0)}} + 0.96 \sqrt{\frac{\mathcal{B}(\eta \eta')}{\mathcal{B}(\eta' K^0)}} \right] \end{aligned}$$

- No best estimate for $\xi_{\eta' K_S}$ yet. My guess $\xi_{\eta' K_S} \sim O(\lambda^2)$
- Numerically $\hat{\xi}_{\eta' K_S} < 0.36$

Bound from charged mode

Similar relations hold for the charged modes

$$a(\eta' K^+) = b(\eta' \pi^+) + s \sqrt{\frac{3}{2}} b(\overline{K^0} K^+)$$

Using experimental data

$$\hat{\xi}_{\eta' K^+} < 0.09$$

- We have $a_{\eta' K^0}^c = a_{\eta' K^+}^c$, but $a_{\eta' K^0}^u \neq a_{\eta' K^+}^u$
- $a_{\eta' K^+}^u$ has a color-allowed tree diagram contribution
- $a_{\eta' K^0}^u$ is color-suppressed
- Dynamical assumption: $|a_{\eta' K^0}^u| \not\approx |a_{\eta' K^+}^u| \Rightarrow \hat{\xi}_{\eta' K^+} < 0.09$

$B \rightarrow \phi K_S$

For PV final state, even more complicated relations

$$\begin{aligned} a(\phi K^0) = & \frac{1}{2} [b(\overline{K^{*0}} K^0) - b(K^{*0} \overline{K^0})] + \frac{1}{2} \sqrt{\frac{3}{2}} [cb(\phi\eta) - sb(\phi\eta')] \\ & + \frac{\sqrt{3}}{4} [cb(\omega\eta) - sb(\omega\eta')] - \frac{\sqrt{3}}{4} [cb(\rho^0\eta) - sb(\rho^0\eta')] \\ & + \frac{1}{4} b(\rho^0\pi^0) - \frac{1}{4} b(\omega\pi^0) - \frac{1}{2\sqrt{2}} b(\phi\pi^0) \end{aligned}$$

- No bound on $\hat{\xi}_{\phi K_S}$ with present data
- No best estimate for $\xi_{\phi K_S}$. My best guess $|\xi_{\phi K_S}| \lesssim \lambda^2$
- Charged modes: $a(\phi K^+) = b(\phi\pi^+) + b(\overline{K^{*0}} K^+)$
- Dynamical assumption $|a_{\phi K^0}^u| \not\approx |a_{\phi K^+}^u| \Rightarrow \hat{\xi}_{\phi K_S} < 0.25$

Comments on SU(3)

- Similar analysis for $B \rightarrow K^+ K^- K_S$; more complicated
- SU(3) relations are most useful for simple relations
- SU(3) and U-spin are the same
- Since we use SU(3) there are large, $O(30\%)$, corrections. They can be larger or smaller in specific cases
- SU(3) (U-spin) can be used only to estimate small effects. They cannot be used to determine basic parameters to high accuracy
- Bottom line: Large deviations from the SU(3) bounds are signals for new physics