Theoretical uncertainties / Possible improvements

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WG 1: Rare Processes

- Generalities
- Classifying uncertainties for specific channels
- Some sort of conclusions

(based on my own reflections . . . and discussions with many people . . .)

Theoretical uncertainties in exclusive B decays (generalities)

- Determination of flavor parameters from *inclusive* decays theoretically "clean", e.g.

$$d\Gamma[B \to X_s \gamma] \;=\; |V_{tb}V_{ts}^*|^2 \, |C_7^{\rm eff}|^2 \, \text{[partonic rate]} \, \big(\text{pert. corr.} + \text{non-pert. corrections}\big)$$

i.e. to first approximation the theoretical prediction is free of hadronic uncertainties, and the sensitivity to non-perturbative effects only enters at the level of corrections

(which still may be sizeable)

- in contrast, the analogous *exclusive* observable depends on non-perturbative input already in leading approximation, e.g.

$$\Gamma[B \to K^* \gamma] \;\propto\; |V_{tb}V_{ts}^*|^2 \, |C_7^{\rm eff}|^2 \, |T^{B\to K^*}(0)|^2 \, \big(\text{pert. corr.} + \text{non-pert. corrections}\big)$$

where $T^{B\to K^*}(q^2)$ is the hadronic form factor for a $B \to K^*$ tensor transition.

How to control/improve theoretical uncertainties

- Extract the product (SM parameters) $\times$ (hadronic quantity) from experimental data. Find appropriate ratios where hadronic parameters drop out (to leading approximation).
  $\longrightarrow$ Accuracy limited by control on perturbative and non-perturbative corrections, only!
- Approximate symmetries in QCD (isospin, flavor $SU(3)$, heavy quark spin/flavor . . .)
  $\longrightarrow$ decrease the number of independent hadronic quantities
  $\longleftrightarrow$ increase the possible ratios to be used in phenomenological analyses
- OPE / heavy quark expansion / effective theories (HQET, SCET, ChPT, . . .):
  $\longrightarrow$ systematic framework to calculate perturbative corrections ("QCD factorization")
  $\longrightarrow$ tool to classify non-perturbative corrections in terms of small expansion parameter(s)
- Rely on non-perturbative methods (lattice, sum rules, models . . .)
  $\longrightarrow$ estimate hadronic parameters, and specify systematic theoretical uncertainties
  $\longrightarrow$ use this input to extract SM parameters from a given observable
- Consider "old physics" experiments:
  $\longrightarrow$ "de-bug" non-perturbative methods (e.g. checking lattice results with CLEO-c)

Dominant uncertainties for specific observables

- "type–I": Observables with leading dependence on non-pert. input:

Observable	Accuracy	dominant uncertainty
BR$[B \to \pi\ell\nu, \rho\ell\nu]$	$\sim 40\%$	$F^{B\to\pi(\rho)}$
BR$[B \to K^*\gamma, K^*\ell^+\ell^-]$	$\sim 45\%$	$F^{B\to K^*}$, "non-factorizable" corrections
BR$[B \to \gamma\ell\nu]$	$\sim 60\%$	$F^{B\to\gamma}$, "non-factorizable" corrections
...		

- "type–II": Appropriate ratios of observables with reduced hadronic uncertainties:

Observable	Accuracy	dominant uncertainty
$\mathrm{BR}[B \to K^*\gamma]/\mathrm{BR}[B \to \rho\gamma]$	$\sim 20\%$	$F^{B\to K^*}/F^{B\to\rho}$
$A_{\rm CP}(B \to \rho\gamma)$	$\sim 20\%$	μ dep., $1/m_b$ corr.
zero in $dA_{\rm FB}[B \to K^*\ell^+\ell^-]$	$\sim 15\%$	μ–dep., m_b, λ_B, $\Lambda_{\rm QCD}$, $1/m_b$ corr.
...		

- "type–III": Observables which are tiny in the SM:

Observable	order of magnitude	suppression factor in SM
BR$[B_s \to \mu^+\mu^-]$	10^{-9}	m_μ^2/m_b^2
$1 - \mathrm{BR}[B \to K\mu^+\mu^-]/\mathrm{BR}[B \to Ke^+e^-]$	$< 1\%$	m_μ^2/m_b^2
$A_{\rm CP}[B \to K^*\gamma]$	$< 1\%$	λ_u/λ_c
...		

Treatment of perturbative QCD effects

- short-distance effects above the scale m_b:
  - ⋆ Wilson coefficients $C_i(m_b)$ in the electroweak Hamiltonian
  - ⋆ Precise higher-orders calculations available (for both SM and many possible extensions)

(latest contribution to 3-loop anomalous dimension for $B \to K^* \ell^+ \ell^-$ by Gambino/Gorbahn/Haisch)

- short-distance effects at the scale m_b:
  - ⋆ OPE / heavy quark expansion in $1/m_b$
  - ⋆ Effective theory formulation (HQET, SCET$_{\rm I}$)

- short-distance effects at the scale $\sqrt{m_b \Lambda}$:
  - ⋆ Effective theory formulation (SCET$_{\rm II}$)
  - ⋆ Specific for exclusive decays where spectator quark participates in hard scattering

Treatment of non-perturbative QCD effects

Challenge: give a reliable estimate of systematic uncertainties!

- Lattice QCD:
  - ⋆ several extrapolations required:
    (continuum limit, quenched approximation, chiral extrapolation, heavy quark extra-(inter-)polation)
  - ⋆ (slow but steady) progress to be expected:
    (improved actions, better understanding of unquenched systematics, computer power, better algorithms)

- QCD sum rules:
  - ⋆ relies on parton-hadron duality
  - ⋆ estimate of uncertainties from variation of threshold parameter s_0 and Borel mass.
  - ⋆ radiative corrections and power-suppressed effects from condensates can be included

- Models: Requires to compare sufficiently many (independent) approaches . . .

Estimated accuracy of some important input parameters

Parameter	Lattice	Sum Rules	Comments
m_b	$\sim 2\%$	$\sim 2\%$	
m_c	$\sim 10\%$	$\sim 10\%$	quenching error!
...			
f_B	$\sim 15\%$	$\sim 15\%$	
$F^{B\to\gamma}(0)$		$\sim 30\%$	photon wave function
$F^{B\to\pi}(0)$ $F^{B\to\rho}(0)$	$\sim 30\%$	$\sim 20\%$	$\leftrightarrow$ experiment
$F^{B\to K^*}/F^{B\to\rho}$	10%	10%	controversial
...			
λ_B	–	30%	recent improvement
$\xi_\pi(0)$	–	?	SR + SCET?
...			

- To improve lattice predictions we need reliable unquenched simulations.
- Possible improvement of systematics in sum rules from cross-check with SCET predictions.
- Note that analogous parameters for higher resonances are barely known.

The cases $B \to \pi \ell \nu$, $B \to \rho \ell \nu$

- Decay rate for $B \to \pi e \nu$ sensitive to $|F_+^{B\to\pi}(q^2)|\,|V_{ub}|$
  - ⋆ For $q^2 \ll m_b^2$ effective theory (HQET+SCET) applies.
    "Soft" (non-factorizable) part of the form factor can be estimated in light-cone sum rules.
  - ⋆ For intermediate q^2 lattice estimates available.
  - ⋆ For $q^2 \sim m_b^2$ effective theory (HQET+ChPT) applies.
    Form factor dominated by nearest B^* pole / requires effective $BB^*\pi$ coupling constant.
  - ⋆ Results often summarized in terms of simple parameterizations:

$$F_+^{B\to\pi}(q^2) := \frac{F_+^{B\to\pi}(0)}{(1-q^2/m_{B*}^2)(1-\alpha\, q^2/m_{B*}^2)}$$

with $F_+^{B\to\pi}(0) = 0.2 - 0.3$ and $\alpha = 0.3 - 0.6$

$\longrightarrow$ Non-perturbative methods should be checked against experimental data!

$\longrightarrow$ Different q^2 regions should be considered separately!

(see CLEO'03)

The case $B \to \pi\pi$, $B \to \pi K$

- in principle sensitive to CKM angles γ and α
- needs information on penguin-to-tree ratio
- dependence on hadronic effects reduced by isospin$/SU(3)_F$ analysis
- absolute values and strong phases of decay amplitudes can be estimated in QCD factorization approach à la BBNS.
  - ⋆ reduced dependence on factorization scales etc.
  - ⋆ limited by non-factorizable $1/m_b$ corrections (which may be "chirally enhanced")
    $\longrightarrow$ predictions for absolute BR's should be taken with care!
  - ⋆ combination with isospin$/SU(3)_F$ analysis
    $\longrightarrow$ still get reliable constraints on unitarity triangle!

$<$ table $>$

Comment on exclusive $b \to s\bar{s}s$ decays

- Branching ratio $B \to K\eta'$ has large uncertainties

  (interference of non-singlet amplitudes, extra contribution from flavor-singlet configurations, . . .)

  $\Rightarrow$ not very useful as a test of SM or NP [Beneke/Neubert]

- Time-dependent CP-asymmetries for $B \to \phi K_s$, $B \to \eta' K_s$ etc. are related to $B \to J/\psi K_s$ up to small Cabbibo-suppressed corrections.

  see e.g. [Grossman et al.]

  $\to$ Current experimental discrepancy cannot be explained by QCD effects only!

$<$ fig $>$

The case $B \to \gamma \ell \nu$

Factorization of short-distance QCD dynamics:

$$\mathcal{A}[B \to \gamma \ell \nu] \propto |V_{ub}| \; f_B \; \underbrace{\int \frac{d\omega}{\omega} \phi_B(\omega)}_{\lambda_B^{-1}} \; T(\omega, q^2) + \text{power corrections}$$

$$= |V_{ub}| \; f_B \; \lambda_B^{-1} \; (1 + \mathcal{O}(\alpha_s)) + \text{power corrections}$$

[Korchemsky/Pirjol/Yan], [Descotes-Genon/Sachrajda], [Lunghi/Pirjol/Wyler], [Bosch et al.]

- uncertainty from value of decay constant f_B — $\sim 15\%$
- uncertainty from inverse moment of B-meson wave function, λ_B^{-1} — $\sim 30\%$
  (see recent sum rule analysis by [Braun/Ivanaov/Korchemsky])
- uncertainty from power-corrections — $\sim 30\%$
  (enhanced by large normalization factor entering the photon wave function) [Ball/Kou]
- uncertainty from truncated perturbative expansion — $\sim 10\%$

$\rightarrow$ Useful to extract λ_B from data (with $> 30\%$ uncertainty)

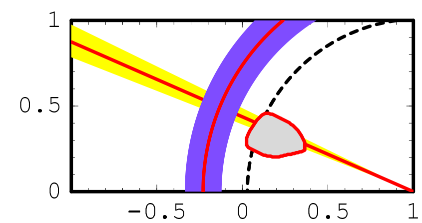
[Important input to estimate factorizable corrections in other observables!]

The cases $B \to K^*\gamma$, $B \to \rho(\omega)\gamma$

(see [Bosch/Buchalla], [Ali/Parkhomenko]), [Beneke/TF/Seidel], [Kagan/Neubert], [Ali/Lunghi])

- branching ratio dominated by form factor uncertainties
  - ⋆ corrections to naive factorization imply smaller $B \to K^*$ form factor than estimated in sum rules (?)
  - ⋆ non-factorizable power-corrections may be important, too

- ratio of $B \to \rho\gamma$ and $B \to K^*\gamma$ can be used to constrain the ratio $R_t = \frac{1}{\lambda}\,\frac{V_{td}}{V_{ts}}$
  - ⋆ requires good knowledge of $SU(3)_F$ breaking in $F^{B\to K^*}/F^{B\to\rho}$
  - ⋆ value may be smaller than usually estimated in sum rules
  - ⋆ at present only upper bound on ratio available from experiment



- isospin asymmetries are less dependent on form factors
  - ⋆ probe $1/m_b$ suppressed weak annihilation contributions
  - ⋆ $\Delta_I[B \to \rho\gamma]$ sensitive to unitarity angle γ
  - ⋆ $\Delta_I[B \to K^*\gamma]$ sensitive to penguin coefficients C_5 and C_6 (but large uncertainties)

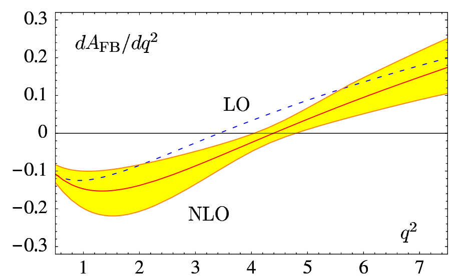
- CP asymmetries:
  - ⋆ $A_{\rm CP}[B \to \rho\gamma]$: uncertainties of order 20%
  - ⋆ $A_{\rm CP}[B \to K^*\gamma]$: tiny in the SM because of CKM hierarchy

The case $B \to K^* \ell^+ \ell^-$

(see [Ali et al.], [Beneke/TF/Seidel], [TF/Matias])

- absolute rates dominated by form factor uncertainties
  - ⋆ model-dependence in $c\bar{c}$ resonance region
  - ⋆ contributions which are not factorizable into form factors

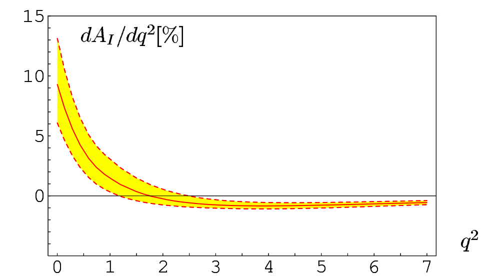
- FB asymmetry in the region $q^2 < 7$ GeV^2:
  - ⋆ form factor dependence drops out to leading approximation
  - ⋆ radiative corrections can be taken into account systematically
  - ⋆ in particular, FB asymmetry zero can be predicted with 15% accuracy
  - ⋆ sensitive to Wilson coefficient C_9 relative to $C_7^{\rm eff}$



scale-dependence from spectator scattering still sizeable

$\longrightarrow$ Include higher-order corrections to hard-scattering kernels!

- Isospin asymmetry in the region $q^2 < 7$ GeV^2:
  - ⋆ requires to take into account $1/m_b$ effects
  - ⋆ isospin effects in form factors themselves ?
  - ⋆ sensitive to penguin coefficients C_3–C_6



The case $B \to K\nu\bar{\nu}$

- Hadronic effects factorize into $B \to K$ form factor(s)

  (analogous situation to $B \to \pi(\rho)\ell\nu$)

  - ⋆ To test SM Wilson coeffficients in this channel,
    form factors have to be determined from elsewhere.
  - ⋆ Ratio $d\Gamma[B \to K\ell^+\ell^-]/d\Gamma[B \to K\nu\bar{\nu}]$ at small q^2
    to first approximation independent of hadronic uncertainties
  - ⋆ but requires sufficient statistics . . . feasible?

The cases $B_s \to \ell^+\ell^-$, $B \to K\ell^+\ell^-$

(see, for instance, Hiller/Krüger, Logan/Nierste)

- $B_s \to \ell^+\ell^-$ is helicity-suppressed by m_l/m_b in the SM
  - ⋆ BR$[B_s \to \mu^+\mu^-]_{\rm SM} =$ few $\times\, 10^{-9}$
  - ⋆ only hadronic uncertainty from f_{B_s} $[\sim 15\%]$
  - ⋆ rate can be significantly enhanced in models with extended Higgs sector and large $\tan\beta$
    $\longrightarrow$ bounds on scalar and pseudoscalar $b \to s\ell^+\ell^-$ operators

- Similar situation for the ratios

$$R_{K^{(*)}} = \frac{\int d\Gamma[B \to K^{(*)}\mu^+\mu^-]}{\int d\Gamma[B \to K^{(*)}e^+e^-]}$$

  - ⋆ $(1 - R_{K^{(*)}})$ is small (of order (m_μ^2/m_b^2)) in the SM (independent of hadronic effects)
  - ⋆ contributions from scalar and pseudoscalar operators can reach several percent

Hadronic uncertainties not really a big issue (yet)

The cases $B \to \gamma\gamma$, $B \to \gamma\ell^+\ell^-$

(see, for instance, Bosch/Buchalla, Descotes-Genon/Sachrajda)

- Depends on same hadronic input (f_B and λ_B) as $B \to \gamma\ell\nu$
  - ⋆ In the heavy quark limit, ratios of amplitudes do not involve physics below scales of order m_b, and are calculable as an expansion in $\alpha_s(m_b)$.
  - ⋆ Unknown (non-factorizable) $1/m_b$ corrections limit theoretical accuracy to $\sim 30\%$
- Direct CP-violation from power-suppressed annihilation graphs

  < Fig. >

  - ⋆ order 10% for $B_d \to \gamma\gamma$ / sensitive to CKM angle γ
  - ⋆ tiny for $B_s \to \gamma\gamma$

  (analogous to $B \to V\gamma$)
- Somewhat challenging because of small BR's (10^{-6} for $B_s \to \gamma\gamma$, 10^{-8} for $B_d \to \gamma\gamma$)

Conclusions

- hadronic uncertainties for individual exclusive rates still around 20-30%

  $\longrightarrow$ not really suited for new physics studies

  $\longrightarrow$ but important tool to test our understanding of "old physics" (i.e. QCD)

- in appropriate *ratios* of exclusive quantities uncertainties can be as small as 10%

  $\longrightarrow$ systematic treatment of perturbative corrections

  $\longrightarrow$ systematic treatment of non-perturbative effects

  $\longrightarrow$ combining effective theory approach, sum rules, lattice . . .

  (may reduce uncertainties in some cases to perhaps 5%)

- Some observables are tiny in the SM ($B_s \to \mu^+\mu^-$, $R_K - 1$, $A_{CP}[B \to K^*\gamma] \ldots$)

  $\longrightarrow$ sensitivity to new physics despite of hadronic uncertainties,
  if SM rates are significantly enhanced.

$B \to \pi\pi$ decay parameters in BBNS approach

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Parameter	Central Value	Dominant Errors	
$\varepsilon_{3/2}$ (%)	$23.9 \pm 4.5 \pm 4.8$	$\pm 3.5\,(m_s)$	$\pm 1.4\,(\mu)$
	$25.7 \pm 4.8 \pm 5.1$	$\pm 3.6\,(m_s)$	$\pm 1.6\,(\alpha_2^K)$
ϕ (deg)	-9.6 ± 3.8	$\pm 3.5\,(m_c)$	$\pm 1.4\,(\alpha_1^K)$
	-10.2 ± 4.1	$\pm 3.7\,(m_c)$	$\pm 1.5\,(\alpha_1^K)$
ε_T (%)	$20.6 \pm 3.5 \pm 4.1$	$\pm 3.2\,(m_s)$	$\pm 0.9\,(\mu)$
	$22.0 \pm 3.6 \pm 4.4$	$\pm 3.3\,(m_s)$	$\pm 0.8\,(\alpha_2^K)$
ϕ_T (deg)	-5.7 ± 4.4	$\pm 3.5\,(m_c)$	$\mp 2.3\,(\mu)$
	-6.2 ± 4.6	$\pm 3.7\,(m_c)$	$\mp 2.2\,(\mu)$
ε_a (%)	$2.0 \pm 0.1 \pm 0.4$	$\pm 0.1\,(m_c)$	$\mp 0.1\,(\mu)$
	$1.9 \pm 0.1 \pm 0.4$	$\pm 0.1\,(m_c)$	—
ϕ_a (deg)	13.6 ± 4.4	$\pm 3.7\,(m_c)$	$\pm 1.7\,(\alpha_1^K)$
	16.6 ± 5.2	$\pm 3.9\,(m_c)$	$\mp 2.8\,(\mu)$
q (%)	$58.8 \pm 6.7 \mp 11.8$	$\pm 6.4\,(R_{\pi K})$	$\pm 1.3\,(\mu)$
ω (deg)	-2.5 ± 2.8	$\pm 1.9\,(\mu)$	$\mp 1.8\,(\alpha_1^K)$
q_C (%)	$8.3 \pm 4.5 \mp 1.7$	$\mp 2.7\,(\lambda_B)$	$\pm 2.3\,(\alpha_1^K)$
	$8.9 \pm 4.9 \mp 1.8$	$\mp 3.1\,(\lambda_B)$	$\pm 2.3\,(\alpha_1^K)$
ω_C (deg)	-60.2 ± 49.5	$\pm 31.7\,(\mu)$	$\mp 27.9\,(\lambda_B)$
	-54.2 ± 44.2	$\pm 29.5\,(\mu)$	$\mp 24.1\,(\lambda_B)$
$\lvert P_{\pi\pi}/T_{\pi\pi}\rvert$ (%)	$28.5 \pm 5.1 \mp 5.7$	$\mp 4.6\,(m_s)$	$\mp 1.8\,(\mu)$
	$25.9 \pm 4.3 \mp 5.2$	$\mp 4.1\,(m_s)$	$\mp 0.8\,(\mu)$
$\arg(P_{\pi\pi}/T_{\pi\pi})$	8.2 ± 3.8	$\mp 3.3\,(m_c)$	$\pm 2.0\,(\mu)$
(deg)	9.0 ± 4.1	$\mp 3.6\,(m_c)$	$\pm 1.8\,(\mu)$

(from Beneke/Buchalla/Neubert/Sachrajda)

$\sin 2\beta$ measurements in $b \to c\bar{c}s$ and $b \to s\bar{s}s$

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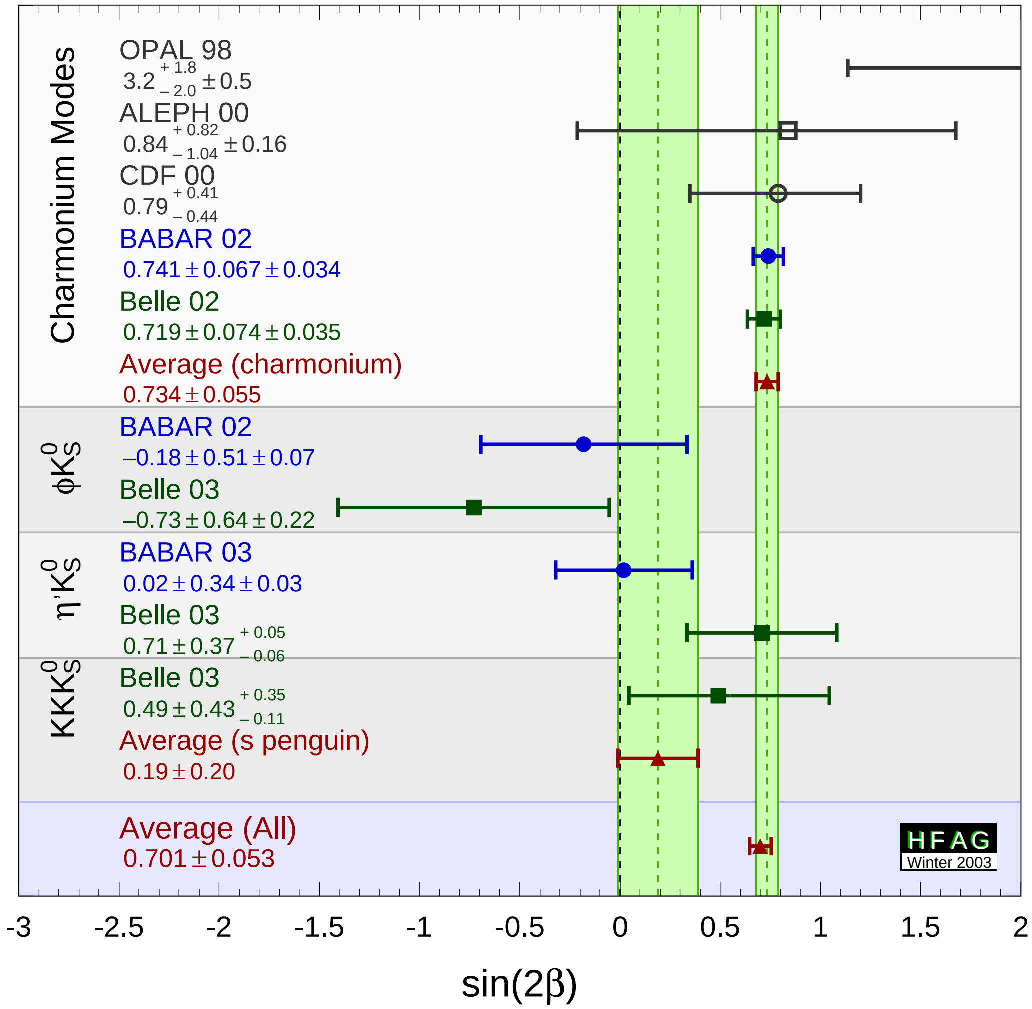

Constraints on unitarity triangle from $\Gamma[B \to \rho\gamma]/\Gamma[B \to K^*\gamma]$

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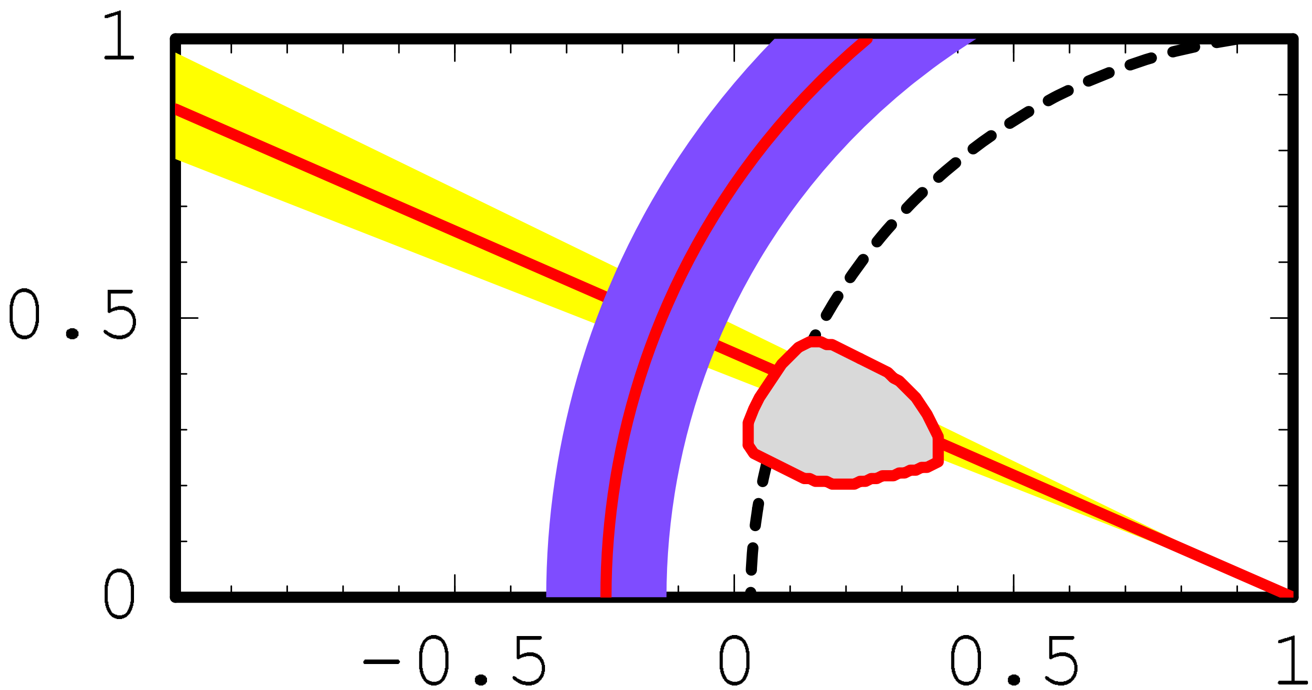

(from [Bosch/Buchalla])

FB asymmetry in $B \to K^* \ell^+ \ell^-$

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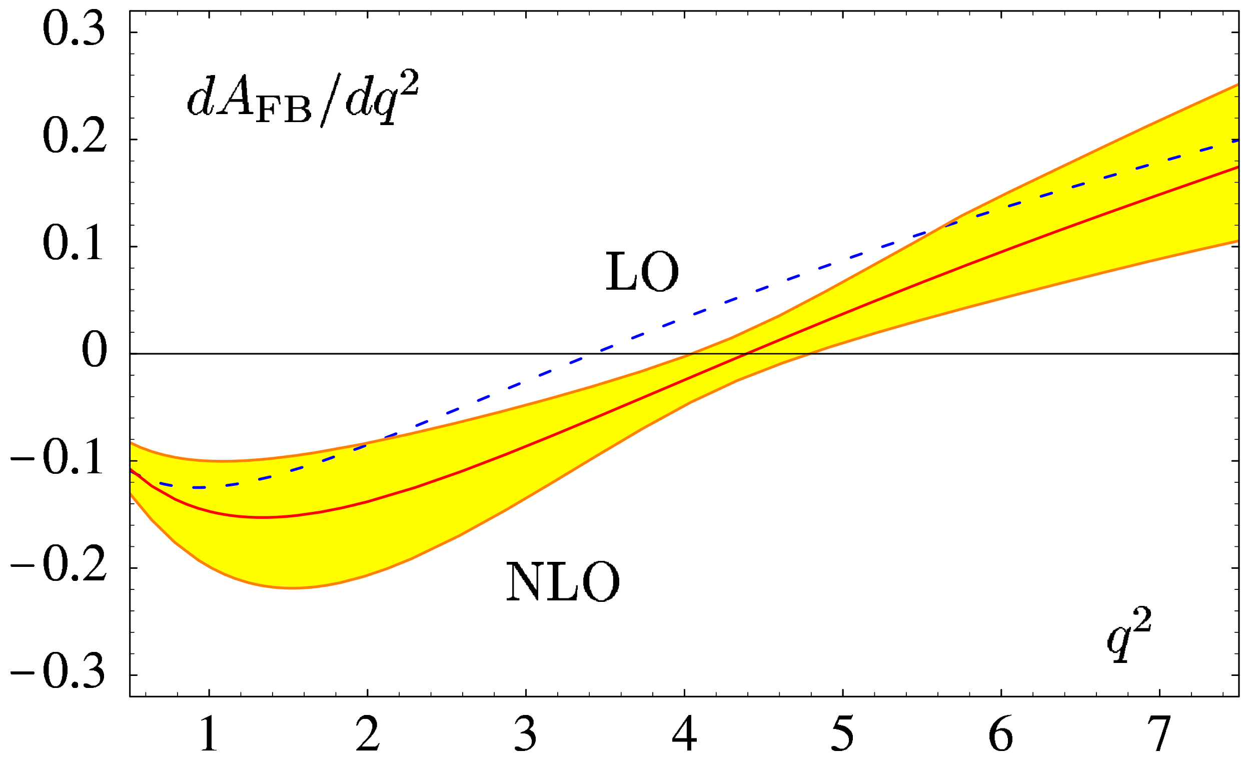

main uncertainties for zero: μ-dep. and $1/m_b$ corrections

(see [Beneke/TF/Seidel])

Isospin asymmetry in $B \to K^* \ell^+ \ell^-$

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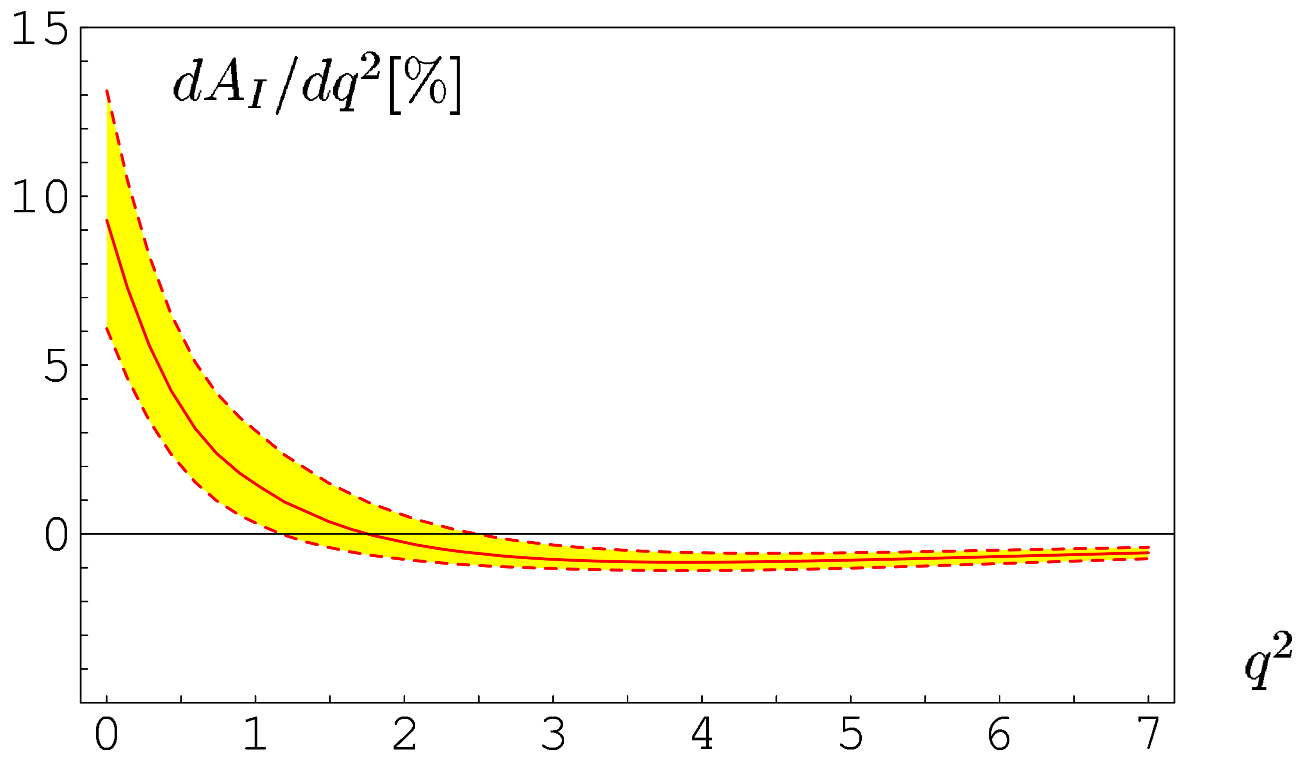

(from [TF/Matias])

CP asymmetry in $B_{d,s} \to \gamma\gamma$

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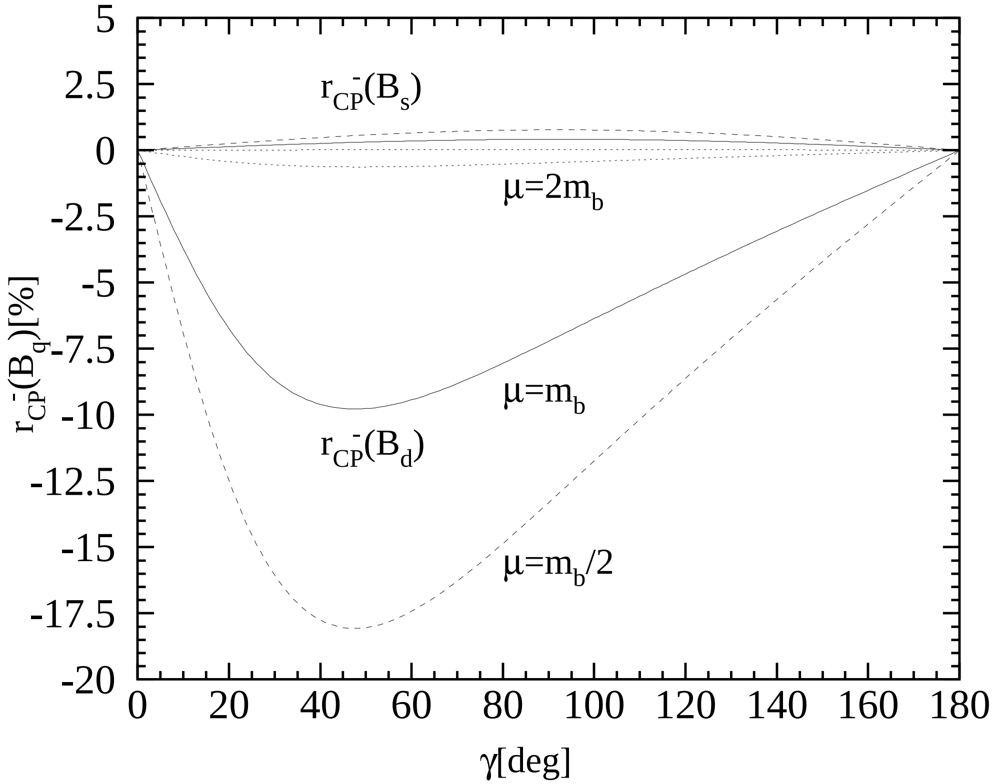

Figure 6: The CP-violating ratios r_{CP}^{-} for decays of neutral B_d and B_s mesons to two photons as a function of the CKM angle γ, each for three values of the renormalization scale $\mu = m_b/2$, m_b and $2m_b$.

(from [Bosch/Buchalla])