Weak phase γ using $B \to K\pi\pi$ modes

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Time dependent measurements of asymmetries of $B_d \to$ CP eigenstates very useful, weak phases extracted, without theoretical uncertainty from modes with single weak phase amplitudes

- Golden Mode $B_d \to J/\psi K_S \Rightarrow \sin 2\beta$
- $B_d \to \pi^+\pi^-$, tree and penguin diagrams, isospin analysis $\Rightarrow \sin 2\alpha$

WHAT ABOUT γ?

- Believed γ cannot be measured using the time dependent techniques
- Other methods developed:

— DK methods, interference of $b \to c$ and $b \to u$ trees, CLEAN

— $K\pi$ methods, interference of $b \to u$ tree and $b \to s$ penguin

Tree is Cabibbo suppressed, Penguin contributions, including Electroweak

Penguins cannot be neglected

Size of the tree and penguin unknown $\longrightarrow$ hadronic uncertainties

Methods using these modes therefore

– Resulted in bounds on γ

– Determine γ, using SU(3), neglect of certain contributions

– Can give good £rst estimates

- Time dependent asymmetry in $K\pi\pi$ decay mode of the $B_d \Rightarrow \gamma$

Look Back at the $K\pi$ modes These modes also obey certain isospin relations,

$$A^{00} + \sqrt{\frac{1}{2}}A^{-+} = A^{0+} + \sqrt{\frac{1}{2}}A^{+0}$$
$$\tilde{A}^{00} + \sqrt{\frac{1}{2}}\tilde{A}^{-+} = \tilde{A}^{0+} + \sqrt{\frac{1}{2}}\tilde{A}^{+0}$$

and form Quadrangles

Unlike the $\pi\pi$ case, the isospin $3/2$ amplitude-which cannot carry a penguin contribution, is infact the diagonal of the quadrangle and not an observable

RESOLUTION Look at the three body $K\pi\pi$ modes

Isolate the $K\pi\pi$ states with $I_{\pi\pi}$ even

$\rightarrow$ two isospin triangle relations between the amplitudes (and corresponding ones for the conjugate amps)

The time dependent asymmetry along with construction of the triangles allows determination of γ

TECHNIQUE

The three body decay modes $B \to K\pi\pi$ described by six independent isospin amplitudes $A(I_t, I_{\pi\pi}, I_f)$

- I_t is the transition isospin, describes the transformation of the weak Hamiltonian under isospin, takes the values 0 and 1 in SM
- $I_{\pi\pi}$ is the isospin of the pion pair, takes the values 0, 1, and 2
- I_f is the £nal isospin and can take the values $1/2$ and $3/2$

Even values of $I_{\pi\pi}$ has the pions in a symmetric state, $\to$ even angular momenta

$I_{\pi\pi}$ odd must be odd under the exchange of two pions

Separation between $I_{\pi\pi} = \mathrm{even}$ and $I_{\pi\pi} = \mathrm{odd}$ possible through a study of the Dalitz plot

Isolate the $K\pi\pi$ states with even isospin, $I_{\pi\pi} = 0, 2$ these are described by the three amplitudes $A(0, 0, \frac{1}{2})$, $A(1, 0, \frac{1}{2})$ and $A(1, 2, \frac{3}{2})$

There are two isospin triangle relations between the amplitudes

(and corresponding ones for the conjugate amps)

$$A(B^+ \to K^0(\pi^+\pi^0)_e) = A(B^0 \to K^0(\pi^+\pi^-)_e) + A(B^0 \to K^0(\pi^0\pi^0)_e)$$

$$A(B^0 \to K^+(\pi^-\pi^0)_e) = A(B^+ \to K^+(\pi^+\pi^-)_e) + A(B^+ \to K^+(\pi^0\pi^0)_e)$$

and also the relation

$$A(B^+ \to K^0(\pi^+\pi^0)_e) = -A(B^0 \to K^+(\pi^-\pi^0)_e)$$

The decay $B(p_B) \to K(k)\pi(p_1)\pi(p_2)$ described in terms of

$s = (p_1 + p_2)^2$, $t = (k + p_1)^2$ and $u = (k + p_2)^2$

States with $I_{\pi\pi} = \text{even}$, symmetric under the exchange $t \leftrightarrow u$

In the $\pi\pi$ rest frame, de£ne θ– angle between direction of the K meson (chosen as the z-axis) with the direction of π.

$\pi\pi$ in even isospin $\to$ even angular momentum $\to$ amplitudes even in $\cos\theta$

States with isospin $I_{\pi\pi} = 1 \to$ odd in $\cos\theta$

Angular analysis $\to$ separate contributions that are $I_{\pi\pi} = 1$, purely $I_{\pi\pi} = \text{even}$ or cross terms between them

Final state $K_S\pi^0\pi^0$ can only exist with $I_{\pi\pi} = \text{even}$

The Amplitudes for the modes $B \to K_S(\pi^+\pi^-)_{\rm e}$, $B \to K_S(\pi^0\pi^0)_{\rm e}$, and $B \to K_S(\pi^+\pi^0)_{\rm e}$ may be written as:

$$
\begin{aligned}
A(B \to K_S(\pi^+\pi^-)_{\rm e}) &\equiv A^{+-} = a^{+-} e^{i\delta_a^{+-}} e^{i\gamma} + b^{+-} e^{i\delta_b^{+-}} \\
A(B \to K_S(\pi^0\pi^0)_{\rm e}) &\equiv A^{00} = a^{00} e^{i\delta_a^{00}} e^{i\gamma} + b^{00} e^{i\delta_b^{00}} \\
A(B \to K_S(\pi^+\pi^0)_{\rm e}) &\equiv A^{+0} = a^{+0} e^{i\delta_a^{+0}} e^{i\gamma} + b^{+0} e^{i\delta_b^{+0}}
\end{aligned}
$$

The magnitudes a^{+-}, b^{+-}, a^{00}, b^{00}, a^{+0} and b^{+0} $\to$ all possible contributions

–Tree, Color-Suppressed, Penguin, Electroweak-Penguin

Annihilation, W-exchange, Penguin-Annihilation

Note, isospin 3/2 amplitude A^{+0} –no contributions from gluonic penguins

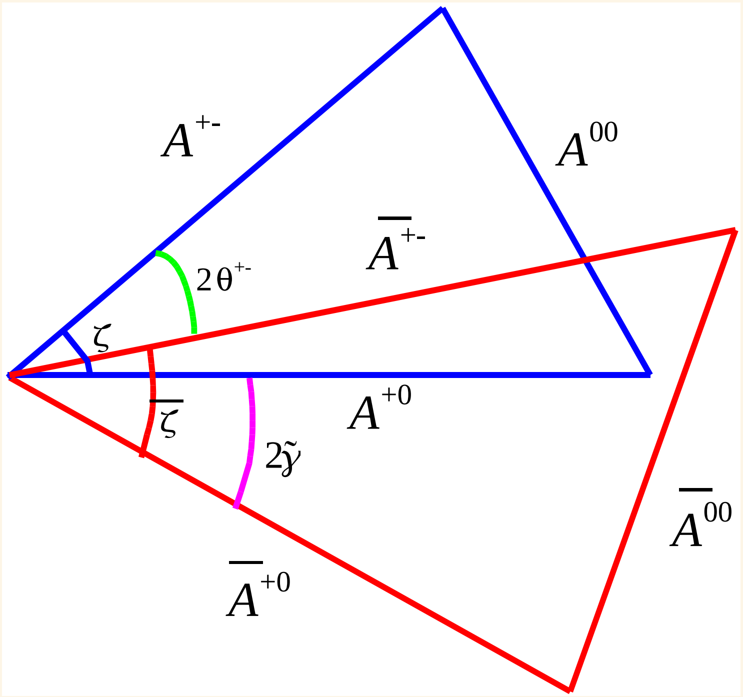
The amplitudes $\overline{A}^{+-}$, $\overline{A}^{00}$, $\overline{A}^{+0}$, for $\bar{B} \to \bar{K}\pi\pi$, similar with $\gamma \to -\gamma$

De£ne,

$\zeta(\bar{\zeta})$: angle between $A^{+-}(\overline{A}^{+-})$ and $A^{+0}(\overline{A}^{+0})$

$2\tilde{\gamma}$: angle between A^{+0} and $\overline{A}^{+0}$

$2\theta^{+-}$: relative phase between A^{+-} and $\overline{A}^{+-}$ (*i.e.* $\arg((A^{+-})^{*}\overline{A}^{+-})$)



$2\theta^{+-}$ obtained from the time dependent CP asymmetry for $B^0(t) \to K_S(\pi^+\pi^-)_e$:

$$\frac{2\,\mathrm{Im}(\lambda^{+-})}{1+|\lambda^{+-}|^2} = y^{+-}\,\sin(2\theta^{+-} - 2\beta), \quad y^f = \sqrt{1-(a^f_{\rm dir})^2}.$$

Note, measurement involves time dependent asymmetry in the partial decay rate $d^2\Gamma^{+-}/d\mathrm{t}d\mathrm{u}$ at a fixed t and u

Events symmetric in $\mathrm{t} \leftrightarrow \mathrm{u}$ need to be selected in the Dalitz plot

With the knowledge of β, $2\theta^{+-}$ may be regarded as an observable

Measurement of six partial decay rates $d^2\Gamma^{+0}/d\mathrm{t}d\mathrm{u}$, $d^2\Gamma^{+-}/d\mathrm{t}d\mathrm{u}$ and $d^2\Gamma^{00}/d\mathrm{t}d\mathrm{u}$ as well as their conjugates at the same t and u as used for θ^{+-} determination, allow us to construct the two isospin triangles with two fold ambiguity

$2\tilde{\gamma}$ related to $2\theta^{+-}$: $\zeta \pm \bar{\zeta} + 2\tilde{\gamma} = 2\theta^{+-}$

Sign ambiguity, same–side or opposite–side orientation of the triangles

$2\tilde{\gamma}$ is known $\Rightarrow$ determine γ

The base of each triangle is again, an isospin $3/2$ amplitude, with no gluonic penguin contribution BUT has an Electroweak Penguin Contribution

Need one additional information : Electroweak penguin operators Q_9 and Q_{10}, Fierz-equivalent to the operators Q_1 and Q_2

Isospin $3/2$ amplitude A^{+0} is symmetric in the two pions $(\pi^+\pi^0)$ $\Rightarrow$ within SM, only the operator $(Q_1 + Q_2)$ contributes, operator $(Q_1 - Q_2)$ does not

The amplitude A^{+0}, has a common strong phase $\delta = \delta_a^{+0} = \delta_b^{+0}$ arising from the same quark operator

Convention: $\delta = 0$

$$A^{+0} = (e^{i\gamma} - \delta_{EW})a^{+0}$$

Therefore, $|A^{+0}| = |\bar{A}^{+0}|$,

and the relative phase between these two is related to γ:

$$\tan\tilde{\gamma} = \frac{\sin\gamma}{\cos\gamma - \delta_{EW}} \, .$$

Since angle $\tilde{\gamma}$ is determined, γ is now calculable

BUT can determine γ without the theoretically computed value of δ_{EW}

γ can be determined cleanly by relying only on the NR observation:

A^{+0} has a single common strong phase

It relies only on– isospin and the operator structures within the SM

Veri£able consequence, Direct Asymmetry for $A^{+0} \equiv A(K^0(\pi^+\pi^0)_{\text{even}}) \to 0$

Isospin violation, tested by comparison of Dalitz plot for $B^+ \to K^0(\pi^+\pi^0)_e$ and $B^0 \to K^+(\pi^-\pi^0)_e$

Using A^{+-}, $\overline{A}^{+-}$, A^{00} and $\overline{A}^{00}$ construct: 7 independent observables

Involve 7 independent parameters: a^{+-}, b^{+-}, a^{00}, b^{00}, a^{+0}, b^{+0}, δ_a^{+-}, δ_b^{+-}, δ_a^{00}, δ_b^{00}, and γ (11)- Number of constraint relations (4)$\longrightarrow$ number of variables=7

SOLUTION

All variables including γ, can be determined purely in terms of observables

To determine γ:

Express all amplitudes and strong phases in terms of observables and γ

- Step I

$$
\begin{aligned}
|a^{+-}|^2 &= \frac{B^{+-}}{2\sin^2\gamma}\left(1 - y^{+-}\cos(2\theta^{+-})\right) , \\
|b^{+-}|^2 &= \frac{B^{+-}}{2\sin^2\gamma}\left(1 - y^{+-}\cos(2\theta^{+-} - 2\gamma)\right) .
\end{aligned}
$$

For a^{+0} and b^{+0}, $B^{+-} \to B^{+0}$, $2\theta^{+-} \to 2\tilde{\gamma}$

a^{00} and b^{+0}, $\quad B^{+-} \to B^{00}$, $\quad 2\theta^{+-} \to 2\theta^{00}$

Here, $2\theta^{00}$, the angle between A^{00} and $\overline{A}^{00}$, need not be measured but can be determined from geometry of the two triangles:

$$\cos(2\theta^{00} - 2\tilde{\gamma}) = \frac{B^{00} - B^{+-} + |A^{+-}||\overline{A}^{+-}|\cos(2\theta^{+-} - 2\tilde{\gamma})}{|A^{00}||\overline{A}^{00}|}$$

- Step II Deﬁne: $\delta^{+-} = \delta_b^{+-} - \delta_a^{+-}$ and $\delta^{00} = \delta_b^{00} - \delta_a^{00}$, δ^{+-} expressed in terms of γ and observables as:

$$\tan\delta^{+-} = \frac{a_{\rm dir}^{+-}\tan\gamma}{1 - y^{+-}[\cos 2\theta^{+-} - \sin 2\theta^{+-}\tan\gamma]},$$

with an analogous expression for $\tan\delta^{00}$.

- Step III Use the isospin relations :

$$
\begin{aligned}
(a^{+-}e^{i\delta_a^{+-}} &+ a^{00}e^{i\delta_a^{00}})e^{\pm i\gamma} + (b^{+-}e^{i\delta_b^{+-}} + b^{00}e^{i\delta_b^{00}}) \\
&= (a^{+0}e^{\pm i\gamma} + b^{+0})
\end{aligned}
$$

These lead to:

$$
\begin{aligned}
\cos\delta_a^{+-} &= \frac{|a^{+0}|^2 + |a^{+-}|^2 - |a^{00}|^2}{2|a^{+0}||a^{+-}|}\,, \\
\cos\delta_a^{00} &= \frac{|a^{+0}|^2 + |a^{00}|^2 - |a^{+-}|^2}{2|a^{+0}||a^{00}|}\,,
\end{aligned}
$$

as well as, the relation,

$$
|b^{+-}|^2 + |b^{00}|^2 + 2b^{+-}b^{00}\cos(\delta_b^{+-} - \delta_b^{00}) = |b^{+0}|^2\,.
$$

This is a relation only in terms of Observables and γ

FEATURES

1. γ can be determined simultaneously for several regions of the Dalitz plot
   $\Rightarrow$ ambiguities in the solution of γ may thereby be removed
2. Having measured γ, can estimate the value of δ_{EW} in terms of observables
   Can verify our understanding of electroweak penguin contributions

FEASABILITY

- Branching ratios of the modes $B^0 \rightarrow K^0\pi^+\pi^-$ and $B^0 \rightarrow K^+\pi^-\pi^0$ have been measured CLEO, BELLE to be around 5×10^{-5}
- Statistically signi£cant contribution in the $K_S\pi^+\pi^-$ mode, from $K^{*+}\pi^-$

Simple isospin analysis $\rightarrow K^{*+}\pi^-$ £nal state *cannot* result in $K^0(\pi^+\pi^-)_o$, but must contribute to $K^0(\pi^+\pi^-)_e$

Preliminary data, based on an integrated luminosity of 43.1fb^{-1}:

$19.1^{+6.8}_{-5.9}$ $K^{*\pm}\pi^\mp$ events in a total of 60.3 ± 11.0 $K^0\pi^+\pi^-$ events

Certainly at the very high luminosity machines, enough $K(\pi^+\pi^-)_e$, to allow a time dependent measurement

Additional $K(\pi^+\pi^-)_e$ events will occur at other regions of Dalitz plot

- While $B^+ \rightarrow K_S\pi^+\pi^0$ not yet been observed, the mode $B^0 \rightarrow K^+\pi^-\pi^0$ has been seen. The two amplitudes are related. If the $K^{*0}\pi^0$ contribution to the $K^+\pi^-\pi^0$ is signi£cant, it must result in $K^+(\pi^-\pi^0)_e$. In future, data from both $B^+ \rightarrow K_S\pi^+\pi^0$ and $B^0 \rightarrow K^+\pi^-\pi^0$ modes could be combined to improve statistics.

CONCLUSIONS:

- Not only determine γ, but can actually also calculate the size of the EWP
- Do not neglect any contributions like annihilation diagrams etc
- Unlike the DK methods, which involve modes with only tree diagrams, since we are here measuring the relative phase of a tree and $b \to s$ penguin, the method will be sensitive to New Physics